# Package 'L1pack' 

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## Type Package

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## $R$ topics documented:

confint.lad ..... 2
ereturns ..... 3
11 fit ..... 3
lad ..... 5
lad.fit ..... 6
lad.fit-methods ..... 7
Laplace ..... 8
LaplaceFit ..... 9
mLaplace ..... 11
simulate.lad ..... 12
vcov.lad ..... 13
WH.Laplace ..... 14
Index ..... 16
confint.lad Confidence intervals from lad models

## Description

Computes confidence intervals for one or more parameters in a fitted model associated to a lad object.

## Usage

```
## S3 method for class 'lad'
confint(object, parm, level = 0.95, ...)
```


## Arguments

| object | a fitted model object. <br> a specification of which parameters are to be given confidence intervals, either <br> a vector of numbers or a vector of names. If missing, all parameters are consid- <br> ered. |
| :--- | :--- |
| level | the confidence level required. |
| $\ldots$ | additional argument(s) for related methods. |

## Details

confint is a generic function. Confidence intervals associated to lad objects are asymptotic, and needs suitable coef and vcov methods to be available.

## Value

A matrix (or vector) with columns giving lower and upper confidence limits for each parameter. These will be labelled as (1-level)/2 and $1-(1-$ level $) / 2$ in $\%$ (by default $2.5 \%$ and $97.5 \%$ ).

## See Also

confint.glm and confint.nls in package MASS.

## Examples

```
fm <- lad(stack.loss ~ ., data = stackloss, method = "BR")
confint(fm) # based on asymptotic normality
```


## Description

Data from the Martin Marietta and American Can companies collected over a period of 5 years on a monthly basis.

## Usage

data(ereturns)

## Format

A data frame with 60 observations on the following 4 variables.

Date the month in which the observations were collected.
am.can excess returns from the American Can company.
m.marietta excess returns from the Martin Marietta company.

CRSP an index for the excess rate returns for the New York stock exchange.

## Source

Butler, R.J., McDonald, J.B., Nelson, R.D., and White, S.B. (1990). Robust and partially adaptive estimation of regression models. The Review of Economics and Statistics 72, 321-327.

## l1fit

Minimum absolute residual (L1) regression

## Description

Performs an L1 regression on a matrix of explanatory variables and a vector of responses.

## Usage

l1fit(x, y, intercept $=$ TRUE, tolerance $=1 \mathrm{e}-07$, print.it $=$ TRUE)

## Arguments

x
y
intercept
tolerance
print.it logical flag. If TRUE, then warnings about non-unique solutions and rank deficiency are given.

## Details

The Barrodale-Roberts algorithm, which is a specialized linear programming algorithm, is used.

## Value

list defining the regression (compare with function lsfit).
coefficients vector of coefficients.
residuals residuals from the fit.
message character strings stating whether a non-unique solution is possible, or if the $x$ matrix was found to be rank deficient.

## References

Barrodale, I., Roberts, F.D.K. (1973). An improved algorithm for discrete L1 linear approximations. SIAM Journal of Numerical Analysis 10, 839-848.

Barrodale, I., Roberts, F.D.K. (1974). Solution of an overdetermined system of equations in the L1 norm. Communications of the ACM 17, 319-320.

Bloomfield, P., Steiger, W.L. (1983). Least Absolute Deviations: Theory, Applications, and Algorithms. Birkhauser, Boston, Mass.

## Examples

l1fit(stack.x, stack.loss)

## Description

This function is used to fit linear models considering Laplace errors.

## Usage

lad(formula, data, subset, na.action, method $=" B R "$, tol $=1 \mathrm{e}-7$, maxiter $=200$, $x=$ FALSE, $y=$ FALSE, contrasts $=$ NULL)

## Arguments

formula an object of class "formula": a symbolic description of the model to be fitted.
data an optional data frame containing the variables in the model. If not found in data, the variables are taken from environment(formula), typically the environment from which lad is called.
subset an optional expression indicating the subset of the rows of data that should be used in the fit.
na.action a function that indicates what should happen when the data contain NAs.
method character string specifying the fitting method to be used; the options are "BR" Barrodale and Roberts' method (the default) and "EM" for an EM algorithm using IRLS.
tol the relative tolerance for the iterative algorithm. Default is tol $=1 \mathrm{e}-7$.
maxiter The maximum number of iterations for the EM method. Default to 200.
$x, y \quad$ logicals. If TRUE the corresponding components of the fit (the model matrix, the response) are returned.
contrasts an optional list. See the contrasts.arg of model.matrix.default.

## Value

An object of class lad representing the linear model fit. Generic function print, show the results of the fit.

The functions print and summary are used to obtain and print a summary of the results. The generic accessor functions coefficients, fitted.values and residuals extract various useful features of the value returned by lad.

## Author(s)

The design was inspired by the R function 1 m .

## References

Barrodale, I., Roberts, F.D.K. (1974). Solution of an overdetermined system of equations in the L1 norm. Communications of the ACM 17, 319-320.
Phillips, R.F. (2002). Least absolute deviations estimation via the EM algorithm. Statistics and Computing 12, 281-285.

## Examples

```
fm <- lad(stack.loss ~ ., data = stackloss, method = "BR")
summary(fm)
```

lad.fit
Fitter functions for least absolute deviation (LAD) regression

## Description

This function is a switcher among various numerical fitting functions (lad.fit.BR, and lad.fit.EM). The argument method does the switching: "BR" for lad.fit.BR, etc. This should usually not be used directly unless by experienced users.

## Usage

lad.fit(x, y, method = "BR", tol = 1e-7, maxiter = 200)

## Arguments

| x | design matrix of dimension $n \times p$. |
| :--- | :--- |
| y | vector of observations of length $n$. |
| method | currently, methods "BR" (default), and "EM" are supported. |
| tol | the relative tolerance for the iterative algorithm. Default is tol $=1 \mathrm{e}-7$. |
| maxiter | The maximum number of iterations for the EM method. Default to 200. |

## Value

a list with components:

| coefficients | a named vector of coefficients. |
| :--- | :--- |
| scale | final scale estimate of the random error. |
| residuals | the residuals, that is response minus fitted values. |
| fitted.values | the fitted values. |
| SAD | the sum of absolute deviations. |
| weights | estimated EM weights. |
| basic | basic observations, that is observations with zero residuals. |
| logLik | the log-likelihood at convergence. |

## See Also

lad.fit. BR, lad.fit.EM.

## Examples

```
x <- cbind(1, stack.x)
fm <- lad.fit(x, stack.loss, method = "BR")
fm
```

lad.fit-methods Fit a least absolute deviation (LAD) regression model

## Description

Fits a linear model using LAD methods, returning the bare minimum computations.

## Usage

lad.fit. $\operatorname{BR}(x, y, t o l=1 e-7)$
lad.fit.EM(x, y, tol = 1e-7, maxiter = 200)

## Arguments

$\mathrm{x}, \mathrm{y} \quad$ numeric vectors or matrices for the predictors and the response in a linear model. Typically, but not necessarily, $x$ will be constructed by one of the fitting functions.
tol the relative tolerance for the iterative algorithm. Default is tol $=1 \mathrm{e}-7$.
maxiter The maximum number of iterations for the EM method. Default to 200.

## Value

The bare bones of a lad object: the coefficients, residuals, fitted values, and some information used by summary.lad.

## See Also

lad, lad.fit, lm

## Examples

```
x <- cbind(1, stack.x)
z <- lad.fit.BR(x, stack.loss)
z
```


## Description

Density, distribution function, quantile function and random generation for the Laplace distribution with location parameter location and scale parameter scale.

## Usage

```
dlaplace(x, location = 0, scale \(=1\), log \(=\) FALSE)
plaplace(q, location \(=0\), scale \(=1\), lower.tail = TRUE, log.p = FALSE)
qlaplace(p, location \(=0\), scale \(=1\), lower.tail = TRUE, log.p = FALSE)
    rlaplace(n, location \(=0\), scale \(=1\) )
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ number of observations.
location location parameter $\mu$, which is the mean.
scale scale parameter $\phi$. Scale must be positive.
log, log.p logical; if TRUE, probabilities $p$ are given as $\log (p)$.
lower.tail logical; if TRUE (default), probabilities are $P(X \leq x)$, otherwise, $P(X>x)$.

## Details

If location or scale are not specified, they assume the default values of 0 and 1 respectively.
The Laplace distribution with location $\mu$ and scale $\phi$ has density

$$
f(x)=\frac{1}{\sqrt{2} \phi} \exp (-\sqrt{2}|x-\mu| / \phi)
$$

where $-\infty<y<\infty,-\infty<\mu<\infty$ and $\phi>0$. The mean is $\mu$ and the variance is $\phi^{2}$.
The cumulative distribution function, assumes the form

$$
F(x)= \begin{cases}\frac{1}{2} \exp (\sqrt{2}(x-\mu) / \phi) & x<\mu \\ 1-\frac{1}{2} \exp (-\sqrt{2}(x-\mu) / \phi) & x \geq \mu\end{cases}
$$

The quantile function, is given by

$$
F^{-1}(p)= \begin{cases}\mu+\frac{\phi}{\sqrt{2}} \log (2 p) & p<0.5 \\ \mu-\frac{\phi}{\sqrt{2}} \log (2(1-p)) & p \geq 0.5\end{cases}
$$

## Value

dlaplace, plaplace, and qlaplace are respectively the density, distribution function and quantile function of the Laplace distribution. rlaplace generates random deviates drawn from the Laplace distribution, the length of the result is determined by $n$.

## Author(s)

Felipe Osorio and Tymoteusz Wolodzko

## References

Kotz, S., Kozubowski, T.J., Podgorski, K. (2001). The Laplace Distributions and Generalizations. Birkhauser, Boston.

Krishnamoorthy, K. (2006). Handbook of Statistical Distributions with Applications, 2nd Ed. Chapman \& Hall, Boca Raton.

## See Also

Distributions for other standard distributions and rmLaplace for the random generation from the multivariate Laplace distribution.

## Examples

```
x <- rlaplace(1000)
## QQ-plot for Laplace data against true theoretical distribution:
qqplot(qlaplace(ppoints(1000)), x, main = "Laplace QQ-plot",
    xlab = "Theoretical quantiles", ylab = "Sample quantiles")
abline(c(0,1), col = "red", lwd = 2)
```

LaplaceFit | Estimation of mean and covariance using the multivariate Laplace dis- |
| :--- |
| tribution |

## Description

Estimates the mean vector and covariance matrix assuming the data came from a multivariate Laplace distribution.

## Usage

LaplaceFit(x, data, subset, na.action, tol = 1e-6, maxiter = 200)

## Arguments

x
data an optional data frame (or similar: see model.frame), used only if x is a formula. By default the variables are taken from environment (formula).
subset an optional expression indicating the subset of the rows of data that should be used in the fitting process.
na.action a function that indicates what should happen when the data contain NAs.
tol the relative tolerance in the iterative algorithm.
maxiter maximum number of iterations. The default is 200.

## Value

A list with class 'LaplaceFit' containing the following components:

| call | a list containing an image of the LaplaceFit call that produced the object. |
| :--- | :--- |
| center | final estimate of the location vector. |
| Scatter | final estimate of the scale matrix. |
| logLik | the log-likelihood at convergence. |
| numIter | the number of iterations used in the iterative algorithm. |
| weights | estimated weights corresponding to the Laplace distribution. |
| distances | estimated squared Mahalanobis distances. |

Generic function print show the results of the fit.

## References

Yavuz, F.G., Arslan, O. (2018). Linear mixed model with Laplace distribution (LLMM). Statistical Papers 59, 271-289.

## See Also

cov

## Examples

```
fit <- LaplaceFit(stack.x)
fit
# covariance matrix
p <- fit$dims[2]
Sigma <- (4 * (p + 1)) * fit$Scatter
Sigma
```

```
mLaplace Multivariate Laplace distribution
```


## Description

These functions provide the density and random number generation from the multivariate Laplace distribution.

## Usage

dmLaplace ( $x$, center $=\operatorname{rep}(0, \operatorname{nrow}($ Scatter $))$, Scatter $=\operatorname{diag}($ length (center $)$ ), log $=$ FALSE)
rmLaplace ( $\mathrm{n}=1$, center $=\operatorname{rep}(0$, nrow(Scatter)), Scatter $=\operatorname{diag}($ length(center)))

## Arguments

$x \quad$ vector or matrix of data.
$\mathrm{n} \quad$ the number of samples requested.
center a $k \times 1$ vector giving the locations.
Scatter a $k \times k$ positive-definite dispersion matrix.
log logical; if TRUE, the logarithm of the density function is returned.

## Details

The multivariate Laplace distribution is a multidimensional extension of the univariate symmetric Laplace distribution. There are multiple forms of the multivariate Laplace distribution. Here, a particular case of the multivarite power exponential distribution introduced by Gomez et al. (1998) is considered.
The multivariate Laplace distribution with location $\boldsymbol{\mu}=$ center and $\boldsymbol{\Sigma}=$ Scatter has density

$$
f(\boldsymbol{x})=\frac{\Gamma(k / 2)}{\pi^{k / 2} \Gamma(k) 2^{k+1}}|\boldsymbol{\Sigma}|^{-1 / 2} \exp \left\{-\frac{1}{2}\left[(\boldsymbol{x}-\boldsymbol{\mu})^{T} \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]^{1 / 2}\right\}
$$

The function rmLaplace is an interface to $C$ routines, which make calls to subroutines from LAPACK. The matrix decomposition is internally done using the Cholesky decomposition. If Scatter is not non-negative definite then there will be a warning message.

## Value

If x is a matrix with $n$ rows, then dmLaplace returns a $n \times 1$ vector considering each row of x as a copy from the multivariate Laplace.
If $n=1$, then rmLaplace returns a vector of the same length as center, otherwise a matrix of $n$ rows of random vectors.

## References

Fang, K.T., Kotz, S., Ng, K.W. (1990). Symmetric Multivariate and Related Distributions. Chapman \& Hall, London.
Gomez, E., Gomez-Villegas, M.A., Marin, J.M. (1998). A multivariate generalization of the power exponential family of distributions. Communications in Statistics - Theory and Methods 27, 589600.

## Examples

```
# dispersion parameters
Scatter <- matrix(c(1,.5,.5,1), ncol = 2)
Scatter
# generate the sample
y <- rmLaplace(n = 2000, Scatter = Scatter)
# scatterplot of a random bivariate Laplace sample with center
# vector zero and scale matrix 'Scatter'
par(pty = "s")
plot(y, xlab = "", ylab = "")
title("bivariate Laplace sample", font.main = 1)
```

simulate.lad Simulate responses from lad models

## Description

Simulate one or more responses from the distribution corresponding to a fitted lad object.

## Usage

\#\# S3 method for class 'lad'
simulate(object, nsim $=1$, seed $=$ NULL, ...)

## Arguments

object an object representing a fitted model.
nsim number of response vectors to simulate. Defaults to 1.
seed an object specifying if and how the random number generator should be initialized ('seeded'). For the "lad" method, either NULL or an integer that will be used in a call to set. seed before simulating the response vectors. If set, the value is saved as the "seed" attribute of the returned value. The default, NULL will not change the random generator state, and return .Random. seed as the "seed" attribute, see 'Value'.
... additional optional arguments.

## Value

For the "lad" method, the result is a data frame with an attribute "seed". If argument seed is NULL, the attribute is the value of . Random. seed before the simulation was started.

## Author(s)

Tymoteusz Wolodzko and Felipe Osorio

## Examples

```
fm <- lad(stack.loss ~ ., data = stackloss)
sm <- simulate(fm, nsim = 4)
```

vcov.lad Calculate variance-covariance matrix from lad models

## Description

Returns the variance-covariance matrix of the main parameters of a fitted model for lad objects. The "main" parameters of model correspond to those returned by coef, and typically do not contain the nuisance scale parameter.

## Usage

```
## S3 method for class 'lad'
vcov(object, ...)
```


## Arguments

object
an object representing a fitted model.
... additional arguments for method functions.

## Value

A matrix of the estimated covariances between the parameter estimates in the linear regression model. This should have row and column names corresponding to the parameter names given by the coef method.

WH.Laplace Wilson-Hilferty transformation

## Description

Returns the Wilson-Hilferty transformation for multivariate Laplace deviates.

## Usage

WH.Laplace(x, center, Scatter)

## Arguments

x
object of class 'LaplaceFit' from which is extracted the estimated Mahalanobis distances of the fitted model. Also $x$ can be a vector or matrix of data with, say, $p$ columns.
center mean vector of the distribution or data vector of length $p$. Not required if x have class 'LaplaceFit'.

Scatter Scatter matrix ( $p$ by $p$ ) of the distribution. Not required if x have class 'LaplaceFit' .

## Details

Let $T=D /(2 p)$ be a Gamma distributed random variable, where $D^{2}$ denotes the squared Mahalanobis distance defined as

$$
D^{2}=(\boldsymbol{x}-\boldsymbol{\mu})^{T} \boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})
$$

Thus, the Wilson-Hilferty transformation is given by

$$
z=\frac{T^{1 / 3}-\left(1-\frac{1}{9 p}\right)}{\left(\frac{1}{9 p}\right)^{1 / 2}}
$$

and $z$ is approximately distributed as a standard normal distribution. This is useful, for instance, in the construction of QQ-plots.

## References

Osorio, F., Galea, M., Henriquez, C., Arellano-Valle, R. (2023). Addressing non-normality in multivariate analysis using the $t$-distribution. AStA Advances in Statistical Analysis 107, 785-813.

Terrell, G.R. (2003). The Wilson-Hilferty transformation is locally saddlepoint. Biometrika 90 , 445-453.

Wilson, E.B., Hilferty, M.M. (1931). The distribution of chi-square. Proceedings of the National Academy of Sciences of the United States of America 17, 684-688.

## Examples

```
Scatter <- matrix(c(1,.5,.5,1), ncol = 2)
Scatter
# generate the sample
y <- rmLaplace(n = 500, Scatter = Scatter)
fit <- LaplaceFit(y)
z <- WH.Laplace(fit)
par(pty = "s")
qqnorm(z, main = "Transformed distances QQ-plot")
abline(c(0,1), col = "red", lwd = 2)
```


## Index

```
* datagen
    simulate.lad, 12
* datasets
    ereturns, 3
* distribution
    Laplace, 8
    mLaplace, 11
* models
    confint.lad, 2
    LaplaceFit,9
    simulate.lad, 12
    vcov.lad, 13
* multivariate
        LaplaceFit, }
        mLaplace, 11
        WH.Laplace, 14
* regression
    l1fit, 3
    lad,5
    lad.fit,6
    lad.fit-methods,7
.Random. seed, 12, 13
coef, 2, 13
confint.glm, 2
confint.lad, 2
confint.nls, 2
cov, 10
Distributions,9
dlaplace (Laplace), 8
dmLaplace (mLaplace), 11
ereturns, 3
l1fit,3
lad, 5,7
lad.fit,6,7
lad.fit-methods, }
lad.fit.BR, 6,7
```

```
lad.fit.BR(lad.fit-methods), }
lad.fit.EM, 6, 7
lad.fit.EM(lad.fit-methods), 7
Laplace, 8
LaplaceFit,9
list,6
lm, 5, 7
lsfit,4
mLaplace, 11
model.frame, 10
plaplace (Laplace), 8
qlaplace (Laplace), 8
rlaplace (Laplace), 8
rmLaplace, }
rmLaplace (mLaplace), 11
simulate.lad, 12
vcov, 2
vcov.lad, 13
WH.Laplace, 14
```

