Minimum Volume Ellipsoids

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September 4, 2017

The problem of finding the ellipsoid of minimum volume containing a set of points $\mathbf{v}_1, ..., \mathbf{v}_n$ is stated as the following optimization problem ([1])

$$\begin{array}{ll} \underset{\mathbf{B},\ \mathbf{d}}{\text{maximize}} & log\ det(\mathbf{B}) \\ \text{subject to} & \\ & ||\mathbf{Bx} + \mathbf{d}|| \quad \leq \quad 1, \quad \forall\ ]vex\ \in\ [\mathbf{v}_1, ..., \mathbf{v}_n] \end{array}$$

The function `minelips` takes as input an $n \times p$ matrix $\mathbf{V}$ containing the points around which we would like to find the minimum volume ellipsoid, and returns the input variables necessary to solve the problem using `sqlp`.

```
R> out <- minelips(V)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b
R> OPTIONS <- out$OPTIONS

R> sqlp(blk,At,C,b,OPTIONS)
```

Numerical Example

We consider a small point configuration of size 25 in two dimensions.

```
R> data(Vminelips)

          V1     V2
 [1,]  1.371 -0.430
 [2,] -0.565 -0.257
 [3,]  0.363 -1.763
 [4,]  0.633  0.460
 [5,]  0.404 -0.640
 [6,] -0.106  0.455
 [7,]  1.512  0.705
 [8,] -0.095  1.035
 [9,]  2.018 -0.609
[10,] -0.063  0.505
[11,]  1.305 -1.717
[12,]  2.287 -0.784
[13,] -1.389 -0.851
```

```
[14,] -0.279 -2.414
[15,] -0.133  0.036
[16,]  0.636  0.206
[17,] -0.284 -0.361
[18,] -2.656  0.758
[19,] -2.440 -0.727
[20,]  1.320 -1.368
[21,] -0.307  0.433
[22,] -1.781 -0.811
[23,] -0.172  1.444
[24,]  1.215 -0.431
[25,]  1.895  0.656

R> out <- minelips(Vminelips)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b
R> OPTIONS <- out$OPTIONS

R> out <- sqlp(blk,At,C,b,OPTIONS)
```

Here, the output we are interested in **B** and **d** are stored in the output vector `y`, but not in a straightforward way.

```
R> y <- out$y

           [,1]
[1,] 0.37878339
[2,] 0.48368646
[3,] 0.01425185
[4,] 0.12947058
[5,] 0.17165170

R> p <- ncol(Vminelips)

R> B <- diag(y[1:p])
R> tmp <- p
R> for(k in 1:(p-1)){
R>    B[(k+1):p,k] <- y[tmp + c(1:(p-k))]
R>    B[1,(k+1):p] <- B[(k+1):p,k]
R>    tmp <- tmp + p - k
R> }

R> d <- y[(tmp+1):length(y)]
```

References

[1] Lieven Vandenberghe, Stephen Boyd, and Shao-Po Wu. Determinant maximization with linear matrix inequality constraints. *SIAM journal on matrix analysis and applications*, 19(2):499–533, 1998.