

Finding the Lovasz Number

Adam Rahman

September 4, 2017

The Lovasz Number of a graph \mathbf{G} , denoted $\vartheta(\mathbf{G})$, is the upper bound on the Shannon capacity of the graph ([1]). For an adjacency matrix $\mathbf{B} = [B_{ij}]$ the problem of finding the Lovasz number is given by the following primal SQLP problem

$$\begin{aligned} & \underset{\mathbf{X}}{\text{minimize}} && \text{tr}(\mathbf{C}\mathbf{X}) \\ & \text{subject to} && \text{tr}(\mathbf{X}) = 1 \\ & && X_{ij} = 0 \text{ if } B_{ij} = 1 \\ & && \mathbf{X} \in \mathcal{S}^n \end{aligned}$$

The function `lovasz` takes as input an adjacency matrix \mathbf{B} , and returns the input variables necessary for the Lovasz number to be found using `sqlp`.

```
R> out <- lovasz(B)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> sqlp(blk,At,C,b)
```

Numerical Example

To compute the Lovasz number using `sqlp`, we need only the (weighted) adjacency matrix representing a graph object.

```
R> data(Glovasz)
```

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
[1,]	0	0	0	1	0	0	1	1	0	0
[2,]	0	0	0	1	0	0	1	0	1	1
[3,]	0	0	0	0	0	0	0	1	0	0
[4,]	1	1	0	0	0	0	0	1	0	1
[5,]	0	0	0	0	0	0	1	1	1	1
[6,]	0	0	0	0	0	0	0	0	1	0
[7,]	1	1	0	0	1	0	0	1	1	1
[8,]	1	0	1	1	1	0	1	0	0	0
[9,]	0	1	0	0	1	1	1	0	0	1
[10,]	0	1	0	1	1	0	1	0	1	0

The Lovasz number for the associated graph is the value of the primal objective function. Again, since the objective function was negated to make the primal problem a minimization, we negate the value of the objective function.

```
R> out <- lovasz(Glovasz)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> out <- sqlp(blk,At,C,b)
R> -out$pobj
```

```
[1] 5
```

References

- [1] László Lovász. On the shannon capacity of a graph. *IEEE Transactions on Information theory*, 25(1):1–7, 1979.