Logarithmic Chebyshev Approximation

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For a $p \times n$ ($p > n$) matrix $\mathbf{B}$ and $p \times 1$ vector $\mathbf{f}$, the Logarithmic Chebyshev Approximation problem is stated as the following optimization problem ([1])

$$\begin{array}{ll} \underset{\mathbf{x},\, t}{\text{minimize}} & t \\ \text{subject to} & \\ & 1/t \quad \leq \quad (\mathbf{x}^\mathsf{T}\mathbf{B}_{i\cdot})/\mathbf{f}_i \quad \leq \quad t, \quad i = 1, ..., p \end{array}$$

where $\mathbf{B}_{i\cdot}$ denotes the i^{th} row of the matrix $\mathbf{B}$. Note that we require each element of $\mathbf{B}_{\cdot j}/\mathbf{f}$ to be greater than or equal to 0 for all j.

The function `logcheby` takes as input a matrix `B` and vector `f`, and returns the input variables necessary to solve the Logarithmic Chebyshev Approximation problem using `sqlp`.

```
R> out <- logcheby(B,f)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> sqlp(blk,At,C,b)
```

Numerical Example

As a numerical example, consider the following

```
R> data(Blogcheby)

          V1    V2    V3    V4    V5
 [1,] 9.148 9.040 3.796 6.756 5.816
 [2,] 9.371 1.387 4.358 9.828 1.579
 [3,] 2.861 9.889 0.374 7.595 3.590
 [4,] 8.304 9.467 9.735 5.665 6.456
 [5,] 6.417 0.824 4.318 8.497 7.758
 [6,] 5.191 5.142 9.576 1.895 5.636
 [7,] 7.366 3.902 8.878 2.713 2.337
 [8,] 1.347 9.057 6.400 8.282 0.900
 [9,] 6.570 4.470 9.710 6.932 0.856
[10,] 7.051 8.360 6.188 2.405 3.052
[11,] 4.577 7.376 3.334 0.430 6.674
[12,] 7.191 8.111 3.467 1.405 0.002
```

```
[13,] 9.347 3.881 3.985 2.164 2.086
[14,] 2.554 6.852 7.847 4.794 9.330
[15,] 4.623 0.039 0.389 1.974 9.256
[16,] 9.400 8.329 7.488 7.194 7.341
[17,] 9.782 0.073 6.773 0.079 3.331
[18,] 1.175 2.077 1.713 3.755 5.151
[19,] 4.750 9.066 2.611 5.144 7.440
[20,] 5.603 6.118 5.144 0.016 6.192
```

```
R> data(flogcheby)
```

```
         V1
 [1,] 0.626
 [2,] 0.217
 [3,] 0.217
 [4,] 0.389
 [5,] 0.942
 [6,] 0.963
 [7,] 0.740
 [8,] 0.733
 [9,] 0.536
[10,] 0.002
[11,] 0.609
[12,] 0.837
[13,] 0.752
[14,] 0.453
[15,] 0.536
[16,] 0.537
[17,] 0.001
[18,] 0.356
[19,] 0.612
[20,] 0.829
```

Note that it must be the case that each element of $\mathbf{B}_{\cdot j}/\mathbf{f}$ must be greater than or equal to 0 for every column of $\mathbf{B}$.

```
R> out <- logcheby(Blogcheby, flogcheby)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> out <- sqlp(blk,At,C,b)
```

Here, the outputs of interest are the optimal value of the objective function (which again we need to negagate due to the negation of the objective function), and the vector $\mathbf{X}$, which is stored in the output variable `y`.

```
R> -out$pobj

[1] 23.08812

R> m <- ncol(Blogcheby)
```

```
R> x <- out$y[1:m]

             [,1]
[1,] 0.001106650
[2,] 0.002661286
[3,] 0.001050662
[4,] 0.002180275
[5,] 0.001435069
```

References

[1] Lieven Vandenberghe, Stephen Boyd, and Shao-Po Wu. Determinant maximization with linear matrix inequality constraints. *SIAM journal on matrix analysis and applications*, 19(2):499–533, 1998.