Risk Parity Portfolios with riskParityPortfolio

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Markowitz portfolio

- Let us denote the **returns** of N assets at time t with the vector $\mathbf{r}_t$.
- Suppose that $\mathbf{r}_t$ follows an i.i.d. distribution (not totally accurate but widely adopted) with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$,
- The **portfolio** vector $\mathbf{w}$ denotes the normalized dollar weights of the N assets ($\mathbf{1}^T\mathbf{w} = 1$).
- **Portfolio return** is $r_t^{\text{portf}} = \mathbf{w}^T\mathbf{r}_t$.
- **Markowitz** proposed in his seminar 1952 paper[1] to find a trade-off between the portfolio expected return $\mathbf{w}^T\boldsymbol{\mu}$ and its risk measured by the variance $\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}$:

$$\begin{array}{ll} \underset{\mathbf{w}}{\text{maximize}} & \mathbf{w}^T\boldsymbol{\mu} - \lambda\mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} & \mathbf{w} \geq \mathbf{0}, \quad \mathbf{1}^T\mathbf{w} = 1, \end{array}$$

  where λ is a parameter that controls how risk-averse the investor is.

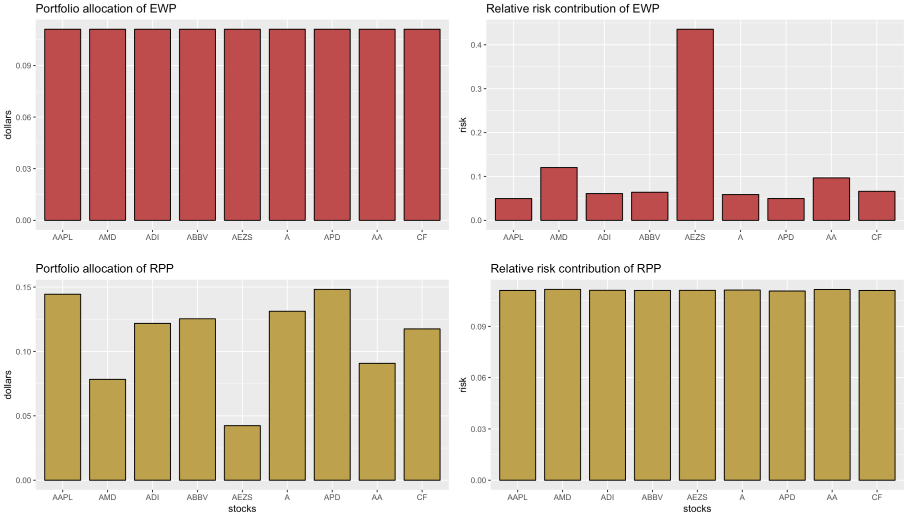
[1] H. Markowitz, "Portfolio selection," *J. Financ.*, vol. 7, no. 1, pp. 77–91, 1952.

History

- **Drawbacks of Markowitz portfolio**: Markowitz's portfolio has been heavily critized for over half a century and has never been fully embraced by practitioners for many reasons:
  - variance is not a good measure of risk,
  - portfolio is highly sensitive to parameter estimation errors,
  - only considers the risk as a whole and ignores the risk diversification.
- **Risk parity** is an approach to portfolio management that focuses on **allocation of risk** rather than allocation of capital.
- Some of its theoretical components were developed in the 1950s and 1960s but the **first risk parity fund, called the "All Weather" fund**, was pioneered by Bridgewater Associates LP in 1996.
- **Some portfolio managers have expressed skepticism** but others point to its performance during the financial crisis of 2007-2008 as an indication of its potential success.

From "dollar" to risk diversification

Equally weighted portfolio (aka uniform portfolio) vs risk parity portfolio:



Risk parity portfolio (RPP)

- From Euler's theorem, the volatility can be decomposed as

$$\sigma(\mathbf{w}) = \sum_{i=1}^{N} \mathrm{RC}_i$$

where RC_i is the **risk contribution (RC)** from the ith asset to the total risk $\sigma(\mathbf{w})$:

$$\mathrm{RC}_i = \frac{w_i (\boldsymbol{\Sigma}\mathbf{w})_i}{\sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}}.$$

- The **risk parity portfolio (RPP)** attemps to "equalize" the risk contributions:

$$\mathrm{RC}_i = \frac{1}{N} \sigma(\mathbf{w}).$$

- More generally, the **risk budgeting portfolio (RBP)** attemps to allocate the risk according to the risk profile determined by the weights $\mathbf{b}$ (with $\mathbf{1}^T\mathbf{b} = 1$ and $\mathbf{b} \geq \mathbf{0}$):

$$\mathrm{RC}_i = b_i \sigma(\mathbf{w}).$$

Solving the RPP

1. **Naive diagonal formulation**: pretend that $\boldsymbol{\Sigma}$ is diagonal and simply use the volatilities $\boldsymbol{\sigma} = \sqrt{\mathrm{diag}(\boldsymbol{\Sigma})}$, obtaining:

$$\mathbf{w} = \frac{\boldsymbol{\sigma}^{-1}}{\mathbf{1}^T \boldsymbol{\sigma}^{-1}}.$$

2. **Vanilla convex formulation**: suppose we only have the constraints $\mathbf{1}^T\mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$, then after some change of variable the problem reduced to solving

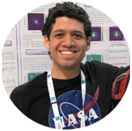
$$\boldsymbol{\Sigma}\mathbf{x} = \mathbf{b}/\mathbf{x}.$$

3. **General nonconvex formulation** (there are many reformulations possible):

$$\begin{array}{ll} \underset{\mathbf{w}}{\text{minimize}} & \sum_{i,j=1}^{N} \left(w_i \left(\boldsymbol{\Sigma}\mathbf{w}\right)_i - w_j \left(\boldsymbol{\Sigma}\mathbf{w}\right)_j \right)^2 - F(\mathbf{w}) \\ \text{subject to} & \mathbf{w} \geq \mathbf{0}, \quad \mathbf{1}^T\mathbf{w} = 1, \quad \mathbf{w} \in \mathcal{W}. \end{array}$$

Package `riskParityPortfolio`

- Some R packages contain functions to compute the RPP, e.g., PortfolioAnalytics, FRAPO, cccp, and FinCovRegularization. But they are based on general-purpose solvers and may not be efficient.
- **riskParityPortfolio** is the first package specifically devised for the computation of different versions of RPP in an efficient way: https://CRAN.R-project.org/package=riskParityPortfolio
- Published on Christmas of 2018 and somehow was well-received by the community (600 downloads in 2 days).
- Authors: Zé Vinícius and Daniel P. Palomar.



Using riskParityPortfolio

- Load Package:

```r
library(riskParityPortfolio)
?riskParityPortfolio  # to get help for the function
```

- The simplest use is for the vanilla RPP:

```r
rpp_vanilla <- riskParityPortfolio(Sigma)
names(rpp_vanilla)
```

```
R>> [1] "w"                 "risk_contribution"
```

```r
print(rpp_vanilla$w, digits = 2)
```

```
R>>  AAPL   AMD   ADI  ABBV  AEZS     A   APD    AA    CF
R>> 0.156 0.068 0.125 0.133 0.045 0.129 0.158 0.085 0.101
```

Using riskParityPortfolio

- Naive diagonal formulation:

```r
rpp_naive <- riskParityPortfolio(Sigma,
                                 formulation = "diag")
```

- Unified nonconvex formulation including expected return in objective and box constraints:

$$
\begin{array}{ll}
\underset{\mathbf{w}}{\text{minimize}} & \sum_{i,j=1}^{N}\left(w_i\left(\boldsymbol{\Sigma}\mathbf{w}\right)_i - w_j\left(\boldsymbol{\Sigma}\mathbf{w}\right)_j\right)^2 - \lambda\,\mathbf{w}^T\boldsymbol{\mu} \\
\text{subject to} & \mathbf{w} \geq \mathbf{0}, \quad \mathbf{1}^T\mathbf{w} = 1, \quad \mathbf{l} \leq \mathbf{w} \leq \mathbf{u}.
\end{array}
$$

```r
rpp_mu <- riskParityPortfolio(Sigma,
                              mu = mu, lmd_mu = 1e-3,
                              w_ub = 0.16)
```

Risk concentration terms

Many formulations included in the package:

$$R(\mathbf{w}) = \sum_{i,j=1}^{N} \left(w_i \left(\boldsymbol{\Sigma} \mathbf{w} \right)_i - w_j \left(\boldsymbol{\Sigma} \mathbf{w} \right)_j \right)^2$$

$$R(\mathbf{w}) = \sum_{i=1}^{N} \left(w_i \left(\boldsymbol{\Sigma} \mathbf{w} \right)_i - \theta \right)^2$$

$$R(\mathbf{w}) = \sum_{i=1}^{N} \left(\frac{w_i (\boldsymbol{\Sigma} \mathbf{w})_i}{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}} - b_i \right)^2$$

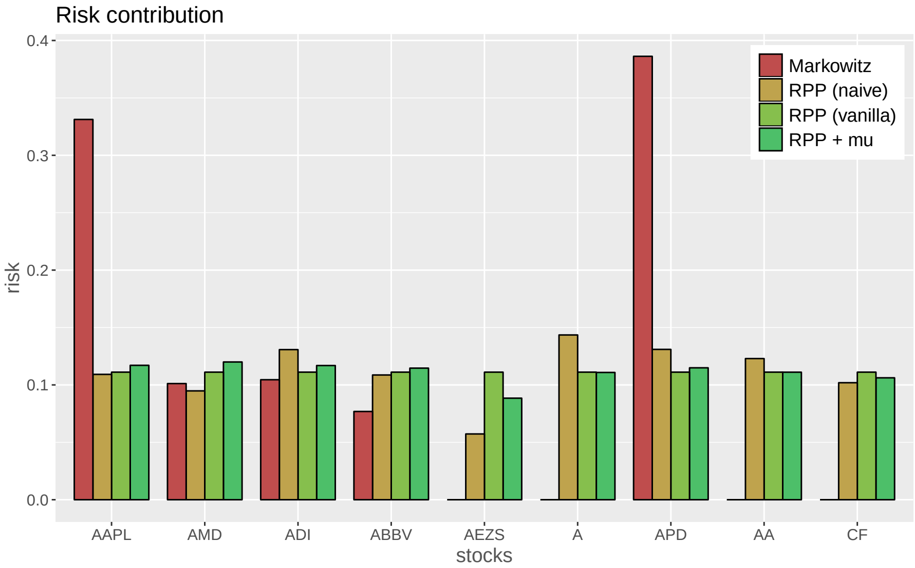
$$R(\mathbf{w}) = \sum_{i,j=1}^{N} \left(\frac{w_i (\boldsymbol{\Sigma} \mathbf{w})_i}{b_i} - \frac{w_j (\boldsymbol{\Sigma} \mathbf{w})_j}{b_j} \right)^2$$

$$R(\mathbf{w}) = \sum_{i=1}^{N} \left(w_i \left(\boldsymbol{\Sigma} \mathbf{w} \right)_i - b_i \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} \right)^2$$

$$R(\mathbf{w}) = \sum_{i=1}^{N} \left(\frac{w_i (\boldsymbol{\Sigma} \mathbf{w})_i}{\sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}} - b_i \sqrt{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}} \right)^2$$

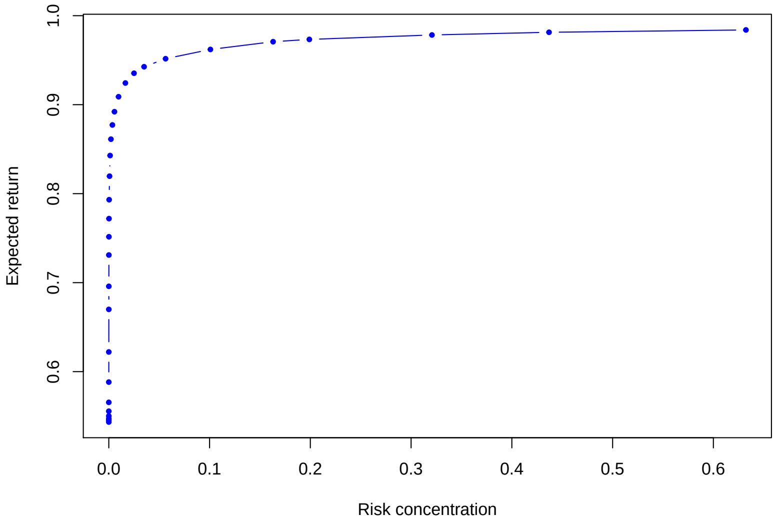
$$R(\mathbf{w}) = \sum_{i=1}^{N} \left(\frac{w_i (\boldsymbol{\Sigma} \mathbf{w})_i}{b_i} - \theta \right)^2$$

Using riskParityPortfolio



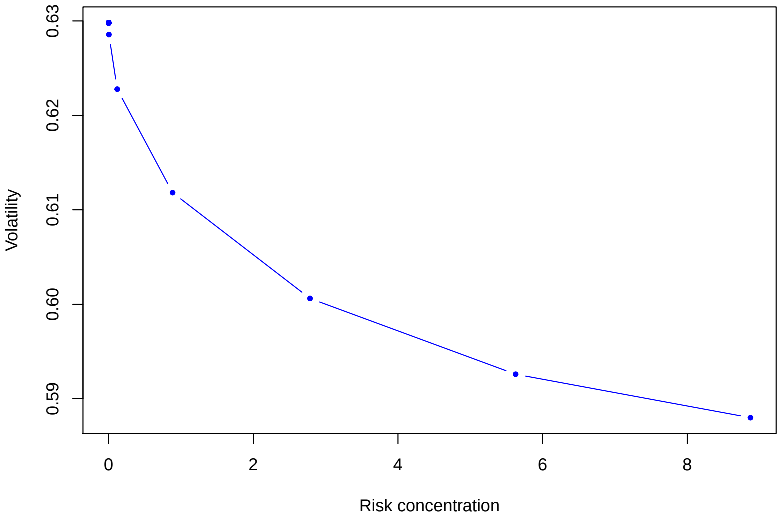
Using `riskParityPortfolio`

Illustration of the **expected return vs risk concentration** trade-off:



Using riskParityPortfolio

Illustration of the **volatility vs risk concentration** trade-off:



References

- Standard textbooks:

T. Roncalli, Introduction to Risk Parity and Budgeting. *CRC Press, 2013.*

E. Qian, *Risk Parity Fundamentals*. CRC Press, 2016.

- Vanilla formulations:

H. Kaya and W. Lee, "Demystifying risk parity," Neuberger Berman, *2012.*

F. Spinu, "An algorithm for computing risk parity weights," *SSRN*, 2013.

T. Griveau-Billion, J.-C. Richard, and T. Roncalli, "A fast algorithm for computing high-dimensional risk parity portfolios," *SSRN*, 2013.

- Unified formulation and advanced algorithms:

Y. Feng and D. P. Palomar, "SCRIP: Successive convex optimization methods for risk parity portfolios design," IEEE Trans. Signal Process., *vol. 63, no. 19, pp. 5285–5300, 2015.*

Thanks

For more information visit:

https://www.danielppalomar.com