Generalized Exponential(GE) distribution:

Parameters : $(\alpha, \lambda) > 0$ and $x \in (0, \infty)$ α shape, λ scale

Cumulative distribution function(cdf):

The distribution function of Generalized exponential(GE) distribution

$$F(x;\alpha,\lambda) = \left\{1 - e^{-\lambda x}\right\}^{\alpha} \quad ; (\alpha,\lambda) > 0, \quad x > 0 \ ;$$

where $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters, respectively.

Probability density function(pdf):

The probability density function

$$f(x;\alpha,\lambda) = \alpha\,\lambda\, e^{-\lambda x}\left\{1 - e^{-\lambda x}\right\}^{\alpha-1} \qquad ; (\alpha,\lambda) > 0, x > 0\,.$$

The Reliability/Survival function(sf):

The reliability/survival function is

$$R(x;\alpha,\lambda) = 1 - \left\{1 - e^{-\lambda x}\right\}^{\alpha} \quad ; (\alpha,\lambda) > 0, \quad x > 0$$

The hazard rate function(hrf):

The reliability/survival function is

$$h(x;\alpha,\lambda) = \frac{\alpha\,\lambda\, e^{-\lambda x}\left\{1 - e^{-\lambda x}\right\}^{\alpha-1}}{1 - \left\{1 - e^{-\lambda x}\right\}^{\alpha}}$$

Indicators:

$$\text{Mean} \quad = \frac{1}{\lambda}\left\{\psi(\alpha+1) - \psi(1)\right\};$$

$$\text{Variance} = \quad \frac{1}{\lambda^2}\left\{\psi'(1) - \psi'(\alpha+1)\right\};$$

$$\text{Mode} \quad = \quad \frac{1}{\lambda}\log\alpha \quad ; \alpha > 1$$

$$\text{Median} \quad = \quad -\frac{1}{\lambda}\log\left\{1 - (0.5)^{1/\alpha}\right\}.$$

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where $\psi(.)$ is the digamma function and $\psi'(.)$ is its derivative.

The Quantile function:

The quantile function is given by

$$x_q = -\frac{1}{\lambda}\log\left\{1 - q^{1/\alpha}\right\} \quad ; 0 < q < 1.$$

Random deviate generation:

Random deviate can be generated by

$$x = -\frac{1}{\lambda}\log\left\{1 - u^{1/\alpha}\right\}$$

where u has the U(0, 1) distribution.

The log-likelihood function:

$$\log \text{density} = \log\alpha + \log\lambda - \lambda\,x + (\alpha - 1)\log\left\{1 - e^{-\lambda\,x}\right\}$$

$$L(\alpha, \lambda) = n\log\alpha + n\log\lambda - \lambda\sum_{i=1}^{n} x_i + (\alpha - 1)\sum_{i=1}^{n}\log\left\{1 - e^{-\lambda\,x_i}\right\}$$

$$\frac{\partial L}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\log\left\{1 - e^{-\lambda\,x_i}\right\} = 0$$

$$\frac{\partial L}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i + (\alpha - 1)\sum_{i=1}^{n}\frac{x_i\,e^{-\lambda\,x_i}}{\left\{1 - e^{-\lambda\,x_i}\right\}} = 0$$

The cumulative hazard function

The cumulative hazard function H(x) defined as

$$H(x) = \int_0^x h(x)\,dx = -\log\left\{1 - F(x)\right\} = -\log\left\{R(x)\right\}$$

can be obtained with the help of pgen.exp()function by choosing arguments *lower.tail=FALSE* and *log.p=TRUE.* i.e.

- *pgen.exp(x, alpha, lambda, lower.tail=FALSE, log.p=TRUE)*

Failure rate average (fra) and Conditional survival function(crf)

Two other relevant functions useful in reliability analysis are failure rate average (fra) and conditional survival function(crf) The failure rate average of X is given by

$$FRA(x) = \frac{H(x)}{x} = \frac{-\log\left\{1 - F(x)\right\}}{x} = \frac{-\log\left\{R(x)\right\}}{x} \qquad , x > 0,$$

where H(x) is the cumulative hazard function. An analysis for FRA(x) on x permits to obtain the IFRA and DFRA classes.

The survival function (s.f.) and the conditional survival of X are defined by

$$R(x) = 1 - F(x)$$

and

$$R(x \mid t) = \frac{R(x+t)}{R(x)}, \quad t > 0,\ x > 0,\ R(\cdot) > 0,$$

respectively, where F(·) is the cdf of X. Similarly to h(x) and FRA(x), the distribution of X belongs to the new better than used (NBU), exponential, or new worse than used (NWU) classes, when $R(x \mid t) < R(x)$, $R(t \mid x) = R(x)$, or $R(x \mid t) > R(x)$, respectively.

References:

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2. Gupta, R. D. and Kundu, D. (1999). Generalized exponential distributions, Australian and New Zealand Journal of Statistics, 41(2), 173 - 188.
3. Gupta, R. D. and Kundu, D. (2001a). Generalized exponential distributions: different methods of estimation, Journal of Statistical Computation and Simulation. 69, 315 - 338.