lmer for SAS PROC MIXED Users

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1 Introduction

The `lmer` function from the `lme4` package for R is used to fit linear mixed-effects models. It is similar in scope to the SAS procedure `PROC MIXED` described in Littell et˜al. (1996).

A file on the SAS Institute web site (http://www.sas.com) contains all the data sets in the book and all the SAS programs used in Littell et˜al. (1996). We have converted the data sets from the tabular representation used for SAS to the `data.frame` objects used by `lmer`. To help users familiar with `SAS PROC MIXED` get up to speed with `lmer` more quickly, we provide transcripts of some `lmer` analyses paralleling the `SAS PROC MIXED` analyses in Littell et˜al. (1996).

In this paper we highlight some of the similarities and differences of `lmer` analysis and `SAS PROC MIXED` analysis.

2 Similarities between lmer and SAS PROC MIXED

Both `SAS PROC MIXED` and `lmer` can fit linear mixed-effects models expressed in the Laird-Ware formulation. For a single level of grouping Laird and Ware (1982) write the n_i-dimensional response vector $\boldsymbol{y}_i$ for the ith experimental

unit as

$$\begin{aligned} \boldsymbol{y}_i &= \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M \\ \boldsymbol{b}_i &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\epsilon}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \end{aligned} \tag{1}$$

where $\boldsymbol{\beta}$ is the p-dimensional vector of *fixed effects*, $\boldsymbol{b}_i$ is the q-dimensional vector of *random effects*, $\boldsymbol{X}_i$ (of size $n_i \times p$) and $\boldsymbol{Z}_i$ (of size $n_i \times q$) are known fixed-effects and random-effects regressor matrices, and $\boldsymbol{\epsilon}_i$ is the n_i-dimensional *within-group error* vector with a spherical Gaussian distribution. The assumption $\mathrm{Var}(\boldsymbol{\epsilon}_i) = \sigma^2\boldsymbol{I}$ can be relaxed using additional arguments in the model fitting.

The basic specification of the model requires a linear model expression for the fixed effects and a linear model expression for the random effects. In `SAS PROC MIXED` the fixed-effects part is specified in the `model` statement and the random-effects part in the `random` statement. In `lmer` the fixed effects and the random effects are both specified as terms in the `formula` argument to `lmer`.

Both `SAS PROC MIXED` and `lmer` allow a mixed-effects model to be fit by maximum likelihood (`method = ml` in SAS) or by maximum residual likelihood, sometimes also called restricted maximum likelihood or REML. This is the default criterion in `SAS PROC MIXED` and in `lmer`. To get ML estimates use the optional argument `REML=FALSE` in the call to `lmer`.

3 Important differences

The output from `PROC MIXED` typically includes values of the Akaike Information Criterion (AIC) and Schwartz's Bayesian Criterion (SBC). These are used to compare different models fit to the same data. The output of the `summary` function applied to the object created by `lmer` also produces values of AIC and BIC but the definitions used in older versions of `PROC MIXED` are different from those used in more recent versions of `PROC MIXED` and in `lmer`. In `lmer` the definitions are such that "smaller is better". In some older versions of `PROC MIXED` the definitions are such that "bigger is better".

When models are fit by REML, the values of AIC, SBC (or BIC) and the log-likelihood can only be compared between models with exactly the same fixed-effects structure. When models are fit by maximum likelihood these criteria can be compared between any models fit to the same data. That is,

these quality-of-fit criteria can be used to evaluate different fixed-effects specifications or different random-effects specifications or different specifications of both fixed effects and random effects.

We encourage developing and testing the model using likelihood ratio tests or the AIC and BIC criteria. Once a form for both the random effects and the fixed effects has been determined, the model can be refit with `REML = TRUE` if the restricted estimates of the variance components are desired. Note that the `update` function provides a convenient way of refitting a model with changes to one or more arguments.

4 Data manipulation

Both `PROC MIXED` and `lmer` work with data in a tabular form with one row per observation. There are, however, important differences in the internal representations of variables in the data.

In SAS a qualitative factor can be stored either as numerical values or alphanumeric labels. When a factor stored as numerical values is used in `PROC MIXED` it is listed in the `class` statement to indicate that it is a factor. In S this information is stored with the data itself by converting the variable to a factor when it is first stored. If the factor represents an ordered set of levels, it should be converted to an `ordered` factor.

For example the SAS code

```
data animal;
 input trait animal y;
 datalines;
1 1 6
1 2 8
1 3 7
2 1 9
2 2 5
2 3 .
;
```

would require that the `trait` and `animal` variables be specified in a class statement in any model that is fit.

In R these data could be read from a file, say `animal.dat`, and converted to factors by

```
animal <- within(read.table("animal.dat", header = TRUE),
                 {
```

```
    trait <- factor(trait)
    animal <- factor(animal)
})
```

In general it is a good idea to check the types of variables in a data frame before working with it. One way of doing this is to apply the function data.class to each variable in turn using the sapply function.

```
> sapply(Animal, data.class)
        Sire          Dam AvgDailyGain
    "factor"     "factor"    "numeric"
> str(Animal)
'data.frame':        20 obs. of  3 variables:
 $ Sire        : Factor w/ 5 levels "1","2","3","4",..: 1 1 1 1 2 2 2 2 3 3 .
 $ Dam         : Factor w/ 2 levels "1","2": 1 1 2 2 1 1 2 2 1 1 ...
 $ AvgDailyGain: num  2.24 1.85 2.05 2.41 1.99 1.93 2.72 2.32 2.33 2.68 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 AvgDailyGain ~ 1 | Sire/Dam
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Sire: logi TRUE
  .. ..$ Dam : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 1
  .. ..$ AvgDailyGain: chr "Average Daily Weight Gain"
  ..$ units       : list()
```

4.1 Unique levels of factors

Designs with nested grouping factors are indicated differently in the two languages. An example of such an experimental design is the semiconductor experiment described in section 2.2 of Littell et˜al. (1996) where twelve wafers are assigned to four experimental treatments with three wafers per treatment. The levels for the wafer factor are 1, 2, and 3 but the wafer factor is only meaningful within the same level of the treatment factor, et. There is nothing associating wafer 1 of the third treatment group with wafer 1 of the first treatment group.

In SAS this nesting of factors is denoted by wafer(et). In S the nesting is written with ~ ET/Wafer and read "wafer within ET". If both levels of nested

factors are to be associated with random effects then this is all you need to know. You would use an expression with a `"/"` in the grouping factor part of the formula in the call to `lmer` object. The random effects term would be either

```
(1 | ET/Wafer)
```

or, equivalently

```
(1 | ET:Wafer) + (1 | ET)
```

In this case, however, there would not usually be any random effects associated with the "experimental treatment" or `ET` factor. The only random effects are at the `Wafer` level. It is necessary to create a factor that will have unique levels for each `Wafer` within each level of `ET`. One way to do this is to assign

```
> Semiconductor <- within(Semiconductor, Grp <- factor(ET:Wafer))
```

after which we could specify a random effects term of `(1 | Grp)`. Alternatively, we can use the explicit term

```
(1 | ET:Wafer)
```

4.2 General approach

As a general approach to importing data into R for mixed-effects analysis you should:

- Create a `data.frame` with one row per observation and one column per variable.
- Use `factor` or `as.factor` to explicitly convert any ordered factors to class `ordered`.
- Use `ordered` or `as.ordered` to explicitly convert any ordered factors to class `ordered`.
- If necessary, use interaction terms to create a factor with unique levels from inner nested factors.
- Plot the data. Plot it several ways. The use of lattice graphics is closely integrated with the `lme4` library. Lattice plots can provide invaluable insight into the structure of the data. Use them.

5 Contrasts

When comparing estimates produced by `SAS PROC MIXED` and by `lmer` one must be careful to consider the contrasts that are used to define the effects of factors. In SAS a model with an intercept and a qualitative factor is defined in terms of the intercept and the indicator variables for all but the last level of the factor. The default behaviour in S is to use the Helmert contrasts for the factor. On a balanced factor these provide a set of orthogonal contrasts. In R the default is the "treatment" contrasts which are almost the same as the SAS parameterization except that they drop the indicator of the first level, not the last level.

When in doubt, check which contrasts are being used with the contrasts function.

To make comparisons easier, you may find it worthwhile to declare

```
> options(contrasts = c(factor = "contr.SAS", ordered = "contr.poly"))
```

at the beginning of your session.

References

Nan˜M. Laird and James˜H. Ware. Random-effects models for longitudinal data. *Biometrics*, 38:963–974, 1982.

Ramon˜C. Littell, George˜A. Milliken, Walter˜W. Stroup, and Russell˜D. Wolfinger. *SAS System for Mixed Models*. SAS Institute, Inc., 1996.

A AvgDailyGain

```
> print(xyplot(adg ~ Treatment | Block, AvgDailyGain, type = c("g", "p", "r")
+         xlab = "Treatment (amount of feed additive)",
+         ylab = "Average daily weight gain (lb.)", aspect = "xy",
+         index.cond = function(x, y) coef(lm(y ~ x))[1]))

> ## compare with output 5.1, p. 178
> (fm1Adg <- lmer(adg ~ (Treatment - 1)*InitWt + (1 | Block), AvgDailyGain))
Linear mixed model fit by REML
Formula: adg ~ (Treatment - 1) * InitWt + (1 | Block)
   Data: AvgDailyGain
```

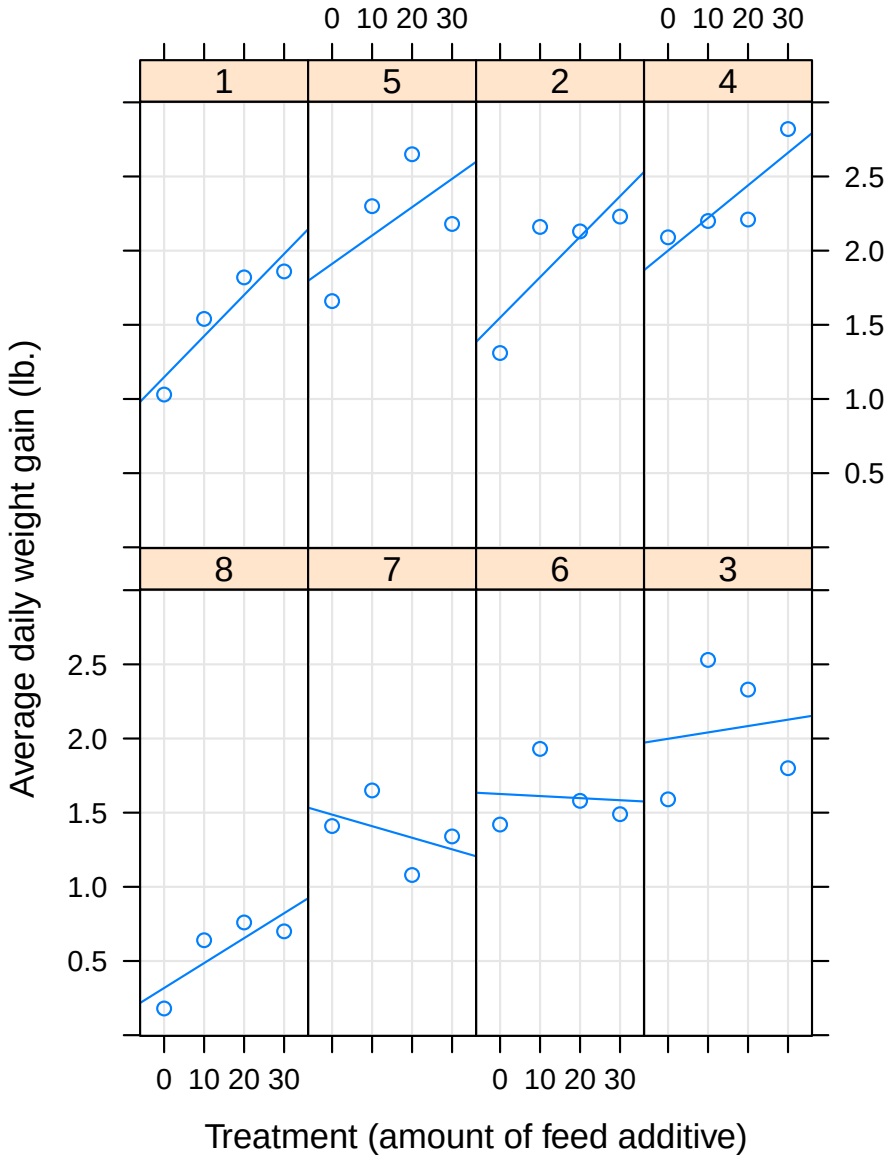

Figure 1: Average daily weight gain

```
   AIC   BIC logLik deviance REMLdev
 85.33 99.98 -32.66     10.1   65.33
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.259312 0.50923
 Residual             0.049429 0.22233
Number of obs: 32, groups: Block, 8

Fixed effects:
                   Estimate Std. Error t value
Treatment0          0.439126   0.711086   0.618
Treatment10         1.426112   0.637543   2.237
Treatment20         0.479620   0.548884   0.874
Treatment30         0.200117   0.775197   0.258
InitWt              0.004448   0.002082   2.137
Treatment0:InitWt  -0.002154   0.002786  -0.773
Treatment10:InitWt -0.003365   0.002515  -1.338
Treatment20:InitWt -0.001082   0.002488  -0.435

Correlation of Fixed Effects:
            Trtmn0 Trtm10 Trtm20 Trtm30 InitWt Tr0:IW T10:IW
Treatment10  0.039
Treatment20  0.080  0.334
Treatment30  0.011  0.097  0.043
InitWt       0.050 -0.032  0.035 -0.967
Trtmnt0:InW -0.640  0.046 -0.024  0.754 -0.780
Trtmnt10:IW -0.021 -0.535 -0.178  0.781 -0.808  0.617
Trtmnt20:IW -0.040 -0.106 -0.512  0.828 -0.856  0.666  0.775
> anova(fm1Adg)   # checking significance of terms
Analysis of Variance Table
                 Df Sum Sq Mean Sq F value
Treatment         4 5.7185 1.42961 28.9224
InitWt            1 0.5495 0.54946 11.1160
Treatment:InitWt  3 0.1381 0.04603  0.9313
> ## common slope model
> (fm2Adg <- lmer(adg ~ InitWt + Treatment + (1 | Block), AvgDailyGain))
Linear mixed model fit by REML
Formula: adg ~ InitWt + Treatment + (1 | Block)
   Data: AvgDailyGain
   AIC  BIC logLik deviance REMLdev
```

```
 50.34 60.6 -18.17    13.62   36.34
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
 Residual             0.05008  0.22379
Number of obs: 32, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
(Intercept)  0.8011046  0.3556585   2.252
InitWt       0.0027797  0.0008334   3.336
Treatment0  -0.5520740  0.1148131  -4.808
Treatment10 -0.0685666  0.1189689  -0.576
Treatment20 -0.0881295  0.1162878  -0.758

Correlation of Fixed Effects:
            (Intr) InitWt Trtmn0 Trtm10
InitWt      -0.844
Treatment0   0.036 -0.224
Treatment10  0.139 -0.340  0.534
Treatment20  0.079 -0.272  0.530  0.545
> anova(fm2Adg)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
InitWt     1 0.51451 0.51451  10.274
Treatment  3 1.52670 0.50890  10.162
> (fm3Adg <- lmer(adg ~ InitWt + Treatment - 1 + (1 | Block), AvgDailyGain))
Linear mixed model fit by REML
Formula: adg ~ InitWt + Treatment - 1 + (1 | Block)
   Data: AvgDailyGain
   AIC  BIC logLik deviance REMLdev
 50.34 60.6 -18.17    13.62   36.34
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.24084  0.49076
 Residual             0.05008  0.22379
Number of obs: 32, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
```

```
InitWt      0.0027797  0.0008334   3.336
Treatment0  0.2490307  0.3776294   0.659
Treatment10 0.7325380  0.3903774   1.876
Treatment20 0.7129751  0.3827661   1.863
Treatment30 0.8011046  0.3556585   2.252

Correlation of Fixed Effects:
            InitWt Trtmn0 Trtm10 Trtm20
Treatment0  -0.863
Treatment10 -0.873  0.957
Treatment20 -0.867  0.957  0.958
Treatment30 -0.844  0.953  0.953  0.953
```

B BIB

```
> print(xyplot(y ~ x | Block, BIB, groups = Treatment, type = c("g", "p"),
+              aspect = "xy", auto.key = list(points = TRUE, space = "right",
+              lines = FALSE)))

> ## compare with Output 5.7, p. 188
> (fm1BIB <- lmer(y ~ Treatment * x + (1|Block), BIB))
Linear mixed model fit by REML
Formula: y ~ Treatment * x + (1 | Block)
   Data: BIB
   AIC   BIC logLik deviance REMLdev
 124.9 136.7 -52.45     93.5   104.9
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.2499  4.2720
 Residual              1.2004  1.0956
Number of obs: 24, groups: Block, 8

Fixed effects:
             Estimate Std. Error t value
(Intercept)  22.36784    3.10182   7.211
Treatment1    4.42949    3.36504   1.316
Treatment2   -0.43737    2.93320  -0.149
Treatment3    6.27864    3.28203   1.913
x             0.44255    0.08706   5.083
Treatment1:x -0.22377    0.10608  -2.109
```

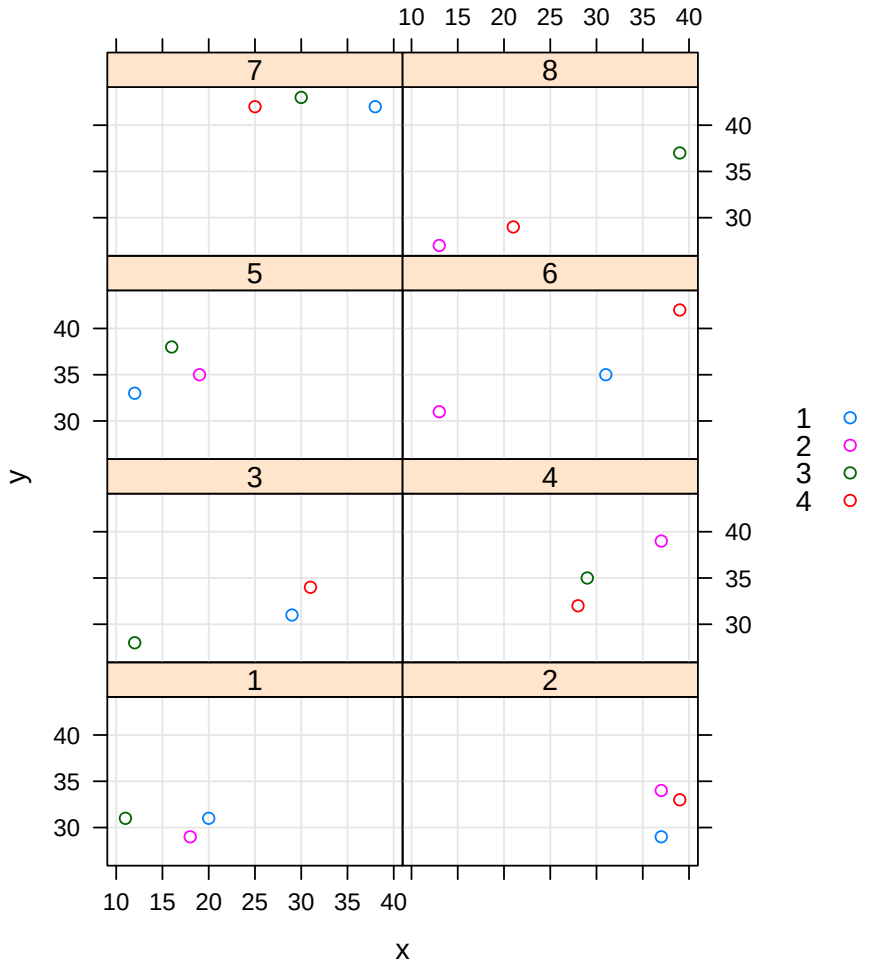

Figure 2: Balanced incomplete block design

```
Treatment2:x  0.05338    0.09714   0.550
Treatment3:x -0.17918    0.11571  -1.549

Correlation of Fixed Effects:
            (Intr) Trtmn1 Trtmn2 Trtmn3 x      Trtm1: Trtm2:
Treatment1  -0.728
Treatment2  -0.778  0.797
Treatment3  -0.796  0.827  0.826
x           -0.859  0.797  0.865  0.886
Treatmnt1:x  0.709 -0.979 -0.774 -0.797 -0.799
Treatmnt2:x  0.722 -0.731 -0.965 -0.763 -0.829  0.729
Treatmnt3:x  0.769 -0.789 -0.790 -0.976 -0.879  0.777  0.748
> anova(fm1BIB)     # strong evidence of different slopes
Analysis of Variance Table
            Df  Sum Sq Mean Sq  F value
Treatment    3  23.447   7.816   6.5110
x            1 136.809 136.809 113.9693
Treatment:x  3  18.427   6.142   5.1169
> ## compare with Output 5.9, p. 193
> (fm2BIB <- lmer(y ~ Treatment + x:Grp + (1|Block), BIB))
Linear mixed model fit by REML
Formula: y ~ Treatment + x:Grp + (1 | Block)
   Data: BIB
   AIC   BIC logLik deviance REMLdev
 115.2 124.6 -49.59    94.09   99.18
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 18.5257  4.3041
 Residual              1.0378  1.0187
Number of obs: 24, groups: Block, 8

Fixed effects:
            Estimate Std. Error t value
(Intercept) 20.94516    2.06230  10.156
Treatment1   5.34145    1.97570   2.704
Treatment2   1.13557    0.71399   1.590
Treatment3   8.18103    1.77010   4.622
x:Grp13      0.23952    0.04296   5.575
x:Grp24      0.48923    0.04412  11.088
```

```
Correlation of Fixed Effects:
           (Intr) Trtmn1 Trtmn2 Trtmn3 x:Gr13
Treatment1 -0.501
Treatment2 -0.431  0.559
Treatment3 -0.527  0.942  0.581
x:Grp13     0.027 -0.663 -0.165 -0.605
x:Grp24    -0.639  0.651  0.452  0.688  0.042
> anova(fm2BIB)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
Treatment  3  23.424   7.808  7.5236
x:Grp      2 154.733  77.367 74.5471
```

C Bond

```
> ## compare with output 1.1 on p. 6
> (fm1Bond <- lmer(pressure ~ Metal + (1|Ingot), Bond))
Linear mixed model fit by REML
Formula: pressure ~ Metal + (1 | Ingot)
   Data: Bond
   AIC BIC logLik deviance REMLdev
 117.8 123  -53.9    115.7   107.8
Random effects:
 Groups   Name        Variance Std.Dev.
 Ingot    (Intercept) 11.448   3.3835
 Residual             10.372   3.2205
Number of obs: 21, groups: Ingot, 7

Fixed effects:
            Estimate Std. Error t value
(Intercept)  71.1000     1.7655   40.27
Metalc       -0.9143     1.7214   -0.53
Metali        4.8000     1.7214    2.79

Correlation of Fixed Effects:
       (Intr) Metalc
Metalc -0.488
Metali -0.488  0.500
> anova(fm1Bond)
Analysis of Variance Table
      Df Sum Sq Mean Sq F value
Metal  2  131.9   65.95  6.3588
```

D Cultivation

```
> str(Cultivation)
'data.frame':        24 obs. of  4 variables:
 $ Block: Factor w/ 4 levels "1","2","3","4": 1 1 1 1 1 1 2 2 2 2 ...
 $ Cult : Factor w/ 2 levels "a","b": 1 1 1 2 2 2 1 1 1 2 ...
 $ Inoc : Factor w/ 3 levels "con","dea","liv": 1 2 3 1 2 3 1 2 3 1 ...
 $ drywt: num  27.4 29.7 34.5 29.4 32.5 34.4 28.9 28.7 33.4 28.7 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 drywt ~ 1 | Block/Cult
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Block: logi TRUE
  .. ..$ Cult : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Cult:Class 'formula' length 2 ~Inoc
  .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ labels      :List of 1
  .. ..$ drywt: chr "Yield"
  ..$ units       : list()
> xtabs(~Block+Cult, Cultivation)
     Cult
Block a b
    1 3 3
    2 3 3
    3 3 3
    4 3 3
> (fm1Cult <- lmer(drywt ~ Inoc * Cult + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML
Formula: drywt ~ Inoc * Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC   BIC logLik deviance REMLdev
 86.49 97.09 -34.24    74.94   68.49
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.20728  1.09876
 Cult     (Intercept) 0.26584  0.51559
 Residual             1.19633  1.09377
Number of obs: 24, groups: Block, 4; Cult, 2
```

```
Fixed effects:
              Estimate Std. Error t value
(Intercept)    33.5250     0.9310   36.01
Inoccon        -5.5000     0.7734   -7.11
Inocdea        -2.8750     0.7734   -3.72
Culta          -0.3750     1.0629   -0.35
Inoccon:Culta   0.2500     1.0938    0.23
Inocdea:Culta  -1.0250     1.0938   -0.94

Correlation of Fixed Effects:
            (Intr) Inoccn Inocde Culta  Incc:C
Inoccon     -0.415
Inocdea     -0.415  0.500
Culta       -0.571  0.364  0.364
Inoccon:Clt  0.294 -0.707 -0.354 -0.515
Inocdea:Clt  0.294 -0.354 -0.707 -0.515  0.500
> anova(fm1Cult)
Analysis of Variance Table
          Df  Sum Sq Mean Sq F value
Inoc       2 118.176  59.088 49.3908
Cult       1   0.656   0.656  0.5487
Inoc:Cult  2   1.826   0.913  0.7631
> (fm2Cult <- lmer(drywt ~ Inoc + Cult + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML
Formula: drywt ~ Inoc + Cult + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC BIC logLik deviance REMLdev
 87.75  96 -36.88     76.9   73.75
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.2128   1.10129
 Cult     (Intercept) 0.2583   0.50824
 Residual             1.1630   1.07842
Number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.6542     0.8691   38.72
Inoccon      -5.3750     0.5392   -9.97
```

```
Inocdea      -3.3875     0.5392   -6.28
Culta        -0.6333     0.8429   -0.75

Correlation of Fixed Effects:
        (Intr) Inoccn Inocde
Inoccon -0.310
Inocdea -0.310  0.500
Culta   -0.485  0.000  0.000
> anova(fm2Cult)
Analysis of Variance Table
     Df  Sum Sq Mean Sq F value
Inoc  2 118.176  59.088 50.8069
Cult  1   0.657   0.657  0.5646
> (fm3Cult <- lmer(drywt ~ Inoc + (1|Block) + (1|Cult), Cultivation))
Linear mixed model fit by REML
Formula: drywt ~ Inoc + (1 | Block) + (1 | Cult)
   Data: Cultivation
   AIC   BIC logLik deviance REMLdev
 87.68 94.75 -37.84    77.32   75.68
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 1.21283  1.10129
 Cult     (Intercept) 0.10364  0.32193
 Residual             1.16299  1.07842
Number of obs: 24, groups: Block, 4; Cult, 2

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.3375     0.7074   47.13
Inoccon      -5.3750     0.5392   -9.97
Inocdea      -3.3875     0.5392   -6.28

Correlation of Fixed Effects:
        (Intr) Inoccn
Inoccon -0.381
Inocdea -0.381  0.500
> anova(fm3Cult)
Analysis of Variance Table
     Df Sum Sq Mean Sq F value
Inoc  2 118.18  59.088  50.807
```

E Demand

```
> ## compare to output 3.13, p. 132
> (fm1Demand <-
+  lmer(log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1|State) + (1|Year),
+       Demand))
Linear mixed model fit by REML
Formula: log(d) ~ log(y) + log(rd) + log(rt) + log(rs) + (1 | State) +      (
   Data: Demand
    AIC    BIC logLik deviance REMLdev
 -224.2 -205.4  120.1   -260.5  -240.2
Random effects:
 Groups   Name        Variance   Std.Dev.
 Year     (Intercept) 0.00026466 0.016268
 State    (Intercept) 0.02950543 0.171771
 Residual             0.00111698 0.033421
Number of obs: 77, groups: Year, 11; State, 7

Fixed effects:
            Estimate Std. Error t value
(Intercept) -1.28386    0.72343  -1.775
log(y)       1.06978    0.10392  10.294
log(rd)     -0.29533    0.05246  -5.629
log(rt)      0.03988    0.02789   1.430
log(rs)     -0.32673    0.11438  -2.856

Correlation of Fixed Effects:
        (Intr) log(y) lg(rd) lg(rt)
log(y)  -0.976
log(rd)  0.383 -0.227
log(rt)  0.077 -0.062 -0.337
log(rs)  0.444 -0.600 -0.270 -0.323
```

F HR

```
> ## linear trend in time
> (fm1HR <- lmer(HR ~ Time * Drug + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML
Formula: HR ~ Time * Drug + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik deviance REMLdev
```

```
 789.6 820.3 -383.8    788.1   767.6
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 60.633   7.7867
          Time        37.784   6.1469   -0.563
 Residual             24.361   4.9357
Number of obs: 120, groups: Patient, 24

Fixed effects:
            Estimate Std. Error t value
(Intercept)  33.9784    10.2826   3.304
Time         -3.1970     3.0850  -1.036
Druga         3.5991     4.2314   0.851
Drugb         7.0912     4.2094   1.685
baseHR        0.5434     0.1161   4.679
Time:Druga   -7.5013     4.3629  -1.719
Time:Drugb   -3.9894     4.3629  -0.914

Correlation of Fixed Effects:
           (Intr) Time   Druga  Drugb  baseHR Tim:Drg
Time       -0.162
Druga      -0.308  0.394
Drugb      -0.244  0.396  0.501
baseHR     -0.957  0.000  0.110  0.041
Time:Druga  0.115 -0.707 -0.557 -0.280  0.000
Time:Drugb  0.115 -0.707 -0.278 -0.560  0.000  0.500
> anova(fm1HR)
Analysis of Variance Table
          Df Sum Sq Mean Sq F value
Time       1 377.77  377.77 15.5072
Drug       2  92.83   46.42  1.9054
baseHR     1 533.01  533.01 21.8799
Time:Drug  2  72.12   36.06  1.4803
> ## remove interaction
> (fm3HR <- lmer(HR ~ Time + Drug + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML
Formula: HR ~ Time + Drug + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik deviance REMLdev
 797.8 822.9 -389.9    791.2   779.8
```

```
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 61.560   7.8460
          Time        40.963   6.4003   -0.571
 Residual             24.361   4.9357
Number of obs: 120, groups: Patient, 24

Fixed effects:
            Estimate Std. Error t value
(Intercept)  36.0471    10.1941   3.536
Time         -7.0273     1.8179  -3.866
Druga        -0.4526     3.5144  -0.129
Drugb         4.9364     3.4879   1.415
baseHR        0.5434     0.1161   4.679

Correlation of Fixed Effects:
       (Intr) Time   Druga  Drugb
Time   -0.096
Druga  -0.297  0.000
Drugb  -0.219  0.000  0.502
baseHR -0.966  0.000  0.132  0.050
> anova(fm3HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq F value
Time    1 362.71  362.71 14.8892
Drug    2  92.84   46.42  1.9055
baseHR  1 533.04  533.04 21.8812
> ## remove Drug term
> (fm4HR <- lmer(HR ~ Time + baseHR + (Time|Patient), HR))
Linear mixed model fit by REML
Formula: HR ~ Time + baseHR + (Time | Patient)
   Data: HR
   AIC   BIC logLik deviance REMLdev
 805.1 824.7 -395.6    794.3   791.1
Random effects:
 Groups   Name        Variance Std.Dev. Corr
 Patient  (Intercept) 63.026   7.9389
          Time        40.963   6.4003   -0.553
 Residual             24.361   4.9357
Number of obs: 120, groups: Patient, 24
```

```
Fixed effects:
            Estimate Std. Error t value
(Intercept)  36.9321     9.9010   3.730
Time         -7.0273     1.8179  -3.866
baseHR        0.5508     0.1175   4.686

Correlation of Fixed Effects:
       (Intr) Time
Time   -0.098
baseHR -0.984  0.000
> anova(fm4HR)
Analysis of Variance Table
       Df Sum Sq Mean Sq F value
Time    1 362.58  362.58  14.884
baseHR  1 534.60  534.60  21.945
```

G Mississippi

```
> ## compare with output 4.1, p. 142
> (fm1Miss <- lmer(y ~ 1 + (1 | influent), Mississippi))
Linear mixed model fit by REML
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
   AIC   BIC logLik deviance REMLdev
 258.4 263.2 -126.2    256.6   252.4
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 63.323   7.9576
 Residual             42.658   6.5313
Number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   21.223      3.429    6.19
> ## compare with output 4.2, p. 143
> (fm1MLMiss <- lmer(y ~ 1 + (1 | influent), Mississippi, method = "ML"))
Linear mixed model fit by maximum likelihood
Formula: y ~ 1 + (1 | influent)
   Data: Mississippi
```

```
   AIC   BIC logLik deviance REMLdev
 262.6 267.4 -128.3    256.6   252.4
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 51.255   7.1592
 Residual             42.697   6.5343
Number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   21.217      3.122   6.796
> ranef(fm1MLMiss)          # BLUP's of random effects on p. 144
$influent
  (Intercept)
1   0.3097985
2  -6.5774724
3  -3.7864007
4   2.8827777
5  -5.8437167
6  13.0150136
> ranef(fm1Miss)            # BLUP's of random effects on p. 142
$influent
  (Intercept)
1   0.3093146
2  -6.7198314
3  -3.8982115
4   2.9463224
5  -6.0133969
6  13.3758027
> VarCorr(fm1Miss)          # compare to output 4.7, p. 148
$influent
            (Intercept)
(Intercept)    63.32337
attr(,"stddev")
(Intercept)
   7.957598
attr(,"correlation")
            (Intercept)
(Intercept)           1

attr(,"sc")
[1] 6.531317
```

```
> ## compare to output 4.8 and 4.9, pp. 150-152
> (fm2Miss <- lmer(y ~ Type + (1 | influent), Mississippi))
Linear mixed model fit by REML
Formula: y ~ Type + (1 | influent)
   Data: Mississippi
   AIC   BIC logLik deviance REMLdev
 244.5 252.6 -117.3    247.5   234.5
Random effects:
 Groups   Name        Variance Std.Dev.
 influent (Intercept) 14.970   3.8691
 Residual             42.514   6.5202
Number of obs: 37, groups: influent, 6

Fixed effects:
            Estimate Std. Error t value
(Intercept)   36.400      4.844   7.514
Type1        -20.800      5.933  -3.506
Type2        -16.462      5.516  -2.984

Correlation of Fixed Effects:
      (Intr) Type1
Type1 -0.816
Type2 -0.878  0.717
> anova(fm2Miss)
Analysis of Variance Table
     Df Sum Sq Mean Sq F value
Type  2 541.62  270.81    6.37
```

H Multilocation

```
> str(Multilocation)
'data.frame':        108 obs. of  7 variables:
 $ obs     : num  3 4 6 7 9 10 12 16 19 20 ...
 $ Location: Factor w/ 9 levels "A","B","C","D",..: 1 1 1 1 1 1 1 1 1 1 ...
 $ Block   : Factor w/ 3 levels "1","2","3": 1 1 1 1 2 2 2 2 3 3 ...
 $ Trt     : Factor w/ 4 levels "1","2","3","4": 3 4 2 1 2 1 3 4 1 2 ...
 $ Adj     : num  3.16 3.12 3.16 3.25 2.71 ...
 $ Fe      : num  7.1 6.68 6.83 6.53 8.25 ...
 $ Grp     : Factor w/ 27 levels "A/1","A/2","A/3",..: 1 1 1 1 2 2 2 2 3 3 ..
 - attr(*, "ginfo")=List of 7
```

```
  ..$ formula     :Class 'formula' length 3 Adj ~ 1 | Location/Block
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups:List of 2
  .. ..$ Location: logi TRUE
  .. ..$ Block   : logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       :List of 1
  .. ..$ Block:Class 'formula' length 2 ~Trt
  .. .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ labels      :List of 1
  .. ..$ Adj: chr "Adjusted yield"
  ..$ units       : list()
> ### Create a Block %in% Location factor
> Multilocation$Grp <- with(Multilocation, Block:Location)
> (fm1Mult <- lmer(Adj ~ Location * Trt + (1|Grp), Multilocation))
Linear mixed model fit by REML
Formula: Adj ~ Location * Trt + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik deviance REMLdev
 86.65 188.6 -5.323   -87.15   10.65
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0056193 0.074962
 Residual             0.0345787 0.185953
Number of obs: 108, groups: Grp, 27

Fixed effects:
             Estimate Std. Error t value
(Intercept)   2.35923    0.11575  20.381
LocationA     0.64930    0.16370   3.966
LocationB     0.06643    0.16370   0.406
LocationC     0.54533    0.16370   3.331
LocationD     0.37413    0.16370   2.285
LocationE     0.55000    0.16370   3.360
LocationF     0.99810    0.16370   6.097
LocationG     0.36057    0.16370   2.203
LocationH     1.01403    0.16370   6.194
Trt1          0.22720    0.15183   1.496
Trt2         -0.00140    0.15183  -0.009
```

```
Trt3            0.42323    0.15183   2.788
LocationA:Trt1 -0.18853    0.21472  -0.878
LocationB:Trt1 -0.27523    0.21472  -1.282
LocationC:Trt1 -0.04000    0.21472  -0.186
LocationD:Trt1 -0.53513    0.21472  -2.492
LocationE:Trt1 -0.26297    0.21472  -1.225
LocationF:Trt1 -0.27153    0.21472  -1.265
LocationG:Trt1  0.20323    0.21472   0.947
LocationH:Trt1 -0.14953    0.21472  -0.696
LocationA:Trt2 -0.09347    0.21472  -0.435
LocationB:Trt2 -0.32273    0.21472  -1.503
LocationC:Trt2  0.08960    0.21472   0.417
LocationD:Trt2 -0.29693    0.21472  -1.383
LocationE:Trt2 -0.30693    0.21472  -1.429
LocationF:Trt2 -0.30993    0.21472  -1.443
LocationG:Trt2 -0.10860    0.21472  -0.506
LocationH:Trt2 -0.33060    0.21472  -1.540
LocationA:Trt3 -0.40247    0.21472  -1.874
LocationB:Trt3 -0.56550    0.21472  -2.634
LocationC:Trt3 -0.12247    0.21472  -0.570
LocationD:Trt3 -0.54840    0.21472  -2.554
LocationE:Trt3 -0.32863    0.21472  -1.531
LocationF:Trt3 -0.46257    0.21472  -2.154
LocationG:Trt3 -0.25297    0.21472  -1.178
LocationH:Trt3 -0.37203    0.21472  -1.733

Correlation of Fixed Effects:
            (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG LoctnH
LocationA   -0.707
LocationB   -0.707  0.500
LocationC   -0.707  0.500  0.500
LocationD   -0.707  0.500  0.500  0.500
LocationE   -0.707  0.500  0.500  0.500  0.500
LocationF   -0.707  0.500  0.500  0.500  0.500  0.500
LocationG   -0.707  0.500  0.500  0.500  0.500  0.500  0.500
LocationH   -0.707  0.500  0.500  0.500  0.500  0.500  0.500  0.500
Trt1        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
Trt2        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
Trt3        -0.656  0.464  0.464  0.464  0.464  0.464  0.464  0.464  0.464
LoctnA:Trt1  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
```

```
LoctnB:Trt1  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt1  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt1  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt1  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
LoctnA:Trt2  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnB:Trt2  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt2  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt2  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt2  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
LoctnA:Trt3  0.464 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnB:Trt3  0.464 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnC:Trt3  0.464 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328 -0.328
LoctnD:Trt3  0.464 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328 -0.328
LoctnE:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328 -0.328
LoctnF:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328 -0.328
LoctnG:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656 -0.328
LoctnH:Trt3  0.464 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.328 -0.656
             Trt1   Trt2   Trt3   LcA:T1 LcB:T1 LcC:T1 LcD:T1 LcE:T1 LcF:T1
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2         0.500
Trt3         0.500  0.500
LoctnA:Trt1 -0.707 -0.354 -0.354
LoctnB:Trt1 -0.707 -0.354 -0.354  0.500
LoctnC:Trt1 -0.707 -0.354 -0.354  0.500  0.500
LoctnD:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500
LoctnE:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500
```

```
LoctnF:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500
LoctnG:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500  0.500
LoctnH:Trt1 -0.707 -0.354 -0.354  0.500  0.500  0.500  0.500  0.500  0.500
LoctnA:Trt2 -0.354 -0.707 -0.354  0.500  0.250  0.250  0.250  0.250  0.250
LoctnB:Trt2 -0.354 -0.707 -0.354  0.250  0.500  0.250  0.250  0.250  0.250
LoctnC:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.500  0.250  0.250  0.250
LoctnD:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.500  0.250  0.250
LoctnE:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.500  0.250
LoctnF:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.500
LoctnG:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.250
LoctnH:Trt2 -0.354 -0.707 -0.354  0.250  0.250  0.250  0.250  0.250  0.250
LoctnA:Trt3 -0.354 -0.354 -0.707  0.500  0.250  0.250  0.250  0.250  0.250
LoctnB:Trt3 -0.354 -0.354 -0.707  0.250  0.500  0.250  0.250  0.250  0.250
LoctnC:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.500  0.250  0.250  0.250
LoctnD:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.500  0.250  0.250
LoctnE:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.500  0.250
LoctnF:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.500
LoctnG:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.250
LoctnH:Trt3 -0.354 -0.354 -0.707  0.250  0.250  0.250  0.250  0.250  0.250
            LcG:T1 LcH:T1 LcA:T2 LcB:T2 LcC:T2 LcD:T2 LcE:T2 LcF:T2 LcG:T2
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2
Trt3
LoctnA:Trt1
LoctnB:Trt1
LoctnC:Trt1
LoctnD:Trt1
LoctnE:Trt1
LoctnF:Trt1
LoctnG:Trt1
LoctnH:Trt1  0.500
LoctnA:Trt2  0.250  0.250
```

```
LoctnB:Trt2  0.250  0.250  0.500
LoctnC:Trt2  0.250  0.250  0.500  0.500
LoctnD:Trt2  0.250  0.250  0.500  0.500  0.500
LoctnE:Trt2  0.250  0.250  0.500  0.500  0.500  0.500
LoctnF:Trt2  0.250  0.250  0.500  0.500  0.500  0.500  0.500
LoctnG:Trt2  0.500  0.250  0.500  0.500  0.500  0.500  0.500  0.500
LoctnH:Trt2  0.250  0.500  0.500  0.500  0.500  0.500  0.500  0.500  0.500
LoctnA:Trt3  0.250  0.250  0.500  0.250  0.250  0.250  0.250  0.250  0.250
LoctnB:Trt3  0.250  0.250  0.250  0.500  0.250  0.250  0.250  0.250  0.250
LoctnC:Trt3  0.250  0.250  0.250  0.250  0.500  0.250  0.250  0.250  0.250
LoctnD:Trt3  0.250  0.250  0.250  0.250  0.250  0.500  0.250  0.250  0.250
LoctnE:Trt3  0.250  0.250  0.250  0.250  0.250  0.250  0.500  0.250  0.250
LoctnF:Trt3  0.250  0.250  0.250  0.250  0.250  0.250  0.250  0.500  0.250
LoctnG:Trt3  0.500  0.250  0.250  0.250  0.250  0.250  0.250  0.250  0.500
LoctnH:Trt3  0.250  0.500  0.250  0.250  0.250  0.250  0.250  0.250  0.250
             LcH:T2 LcA:T3 LcB:T3 LcC:T3 LcD:T3 LcE:T3 LcF:T3 LcG:T3
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2
Trt3
LoctnA:Trt1
LoctnB:Trt1
LoctnC:Trt1
LoctnD:Trt1
LoctnE:Trt1
LoctnF:Trt1
LoctnG:Trt1
LoctnH:Trt1
LoctnA:Trt2
LoctnB:Trt2
LoctnC:Trt2
LoctnD:Trt2
LoctnE:Trt2
```

```
LoctnF:Trt2
LoctnG:Trt2
LoctnH:Trt2
LoctnA:Trt3  0.250
LoctnB:Trt3  0.250  0.500
LoctnC:Trt3  0.250  0.500  0.500
LoctnD:Trt3  0.250  0.500  0.500  0.500
LoctnE:Trt3  0.250  0.500  0.500  0.500  0.500
LoctnF:Trt3  0.250  0.500  0.500  0.500  0.500  0.500
LoctnG:Trt3  0.250  0.500  0.500  0.500  0.500  0.500  0.500
LoctnH:Trt3  0.500  0.500  0.500  0.500  0.500  0.500  0.500  0.500
> anova(fm1Mult)
Analysis of Variance Table
             Df Sum Sq Mean Sq F value
Location      8 6.9474 0.86843 25.1145
Trt           3 1.2217 0.40725 11.7774
Location:Trt 24 0.9966 0.04152  1.2008
> (fm2Mult <- lmer(Adj ~ Location + Trt + (1|Grp), Multilocation))
Linear mixed model fit by REML
Formula: Adj ~ Location + Trt + (1 | Grp)
   Data: Multilocation
 AIC   BIC logLik deviance REMLdev
  22 59.55  3.001   -51.22  -6.001
Random effects:
 Groups   Name        Variance  Std.Dev.
 Grp      (Intercept) 0.0050851 0.07131
 Residual             0.0367154 0.19161
Number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.53296    0.07599   33.33
LocationA    0.47818    0.09752    4.90
LocationB   -0.22443    0.09752   -2.30
LocationC    0.52712    0.09752    5.41
LocationD    0.02902    0.09752    0.30
LocationE    0.32537    0.09752    3.34
LocationF    0.73709    0.09752    7.56
LocationG    0.32098    0.09752    3.29
LocationH    0.80099    0.09752    8.21
```

```
Trt1         0.05834    0.05215    1.12
Trt2        -0.18802    0.05215   -3.61
Trt3         0.08379    0.05215    1.61

Correlation of Fixed Effects:
          (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG LoctnH
LocationA -0.642
LocationB -0.642  0.500
LocationC -0.642  0.500  0.500
LocationD -0.642  0.500  0.500  0.500
LocationE -0.642  0.500  0.500  0.500  0.500
LocationF -0.642  0.500  0.500  0.500  0.500  0.500
LocationG -0.642  0.500  0.500  0.500  0.500  0.500  0.500
LocationH -0.642  0.500  0.500  0.500  0.500  0.500  0.500  0.500
Trt1      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt2      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
Trt3      -0.343  0.000  0.000  0.000  0.000  0.000  0.000  0.000  0.000
          Trt1   Trt2
LocationA
LocationB
LocationC
LocationD
LocationE
LocationF
LocationG
LocationH
Trt1
Trt2       0.500
Trt3       0.500  0.500
> (fm3Mult <- lmer(Adj ~ Location + (1|Grp), Multilocation))
Linear mixed model fit by REML
Formula: Adj ~ Location + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik deviance REMLdev
 31.94 61.44 -4.968   -22.96   9.935
Random effects:
 Groups   Name        Variance   Std.Dev.
 Grp      (Intercept) 3.7983e-14 1.9489e-07
 Residual             5.1642e-02 2.2725e-01
Number of obs: 108, groups: Grp, 27
```

```
Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.52149    0.06560   38.44
LocationA    0.47818    0.09277    5.15
LocationB   -0.22443    0.09277   -2.42
LocationC    0.52712    0.09277    5.68
LocationD    0.02902    0.09277    0.31
LocationE    0.32537    0.09277    3.51
LocationF    0.73709    0.09277    7.95
LocationG    0.32098    0.09277    3.46
LocationH    0.80099    0.09277    8.63

Correlation of Fixed Effects:
          (Intr) LoctnA LoctnB LoctnC LoctnD LoctnE LoctnF LoctnG
LocationA -0.707
LocationB -0.707  0.500
LocationC -0.707  0.500  0.500
LocationD -0.707  0.500  0.500  0.500
LocationE -0.707  0.500  0.500  0.500  0.500
LocationF -0.707  0.500  0.500  0.500  0.500  0.500
LocationG -0.707  0.500  0.500  0.500  0.500  0.500  0.500
LocationH -0.707  0.500  0.500  0.500  0.500  0.500  0.500  0.500
> (fm4Mult <- lmer(Adj ~ Trt + (1|Grp), Multilocation))
Linear mixed model fit by REML
Formula: Adj ~ Trt + (1 | Grp)
   Data: Multilocation
   AIC  BIC logLik deviance REMLdev
 43.51 59.6 -15.75    14.95   31.51
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.110922 0.33305
 Residual             0.036715 0.19161
Number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.86567    0.07395   38.75
Trt1         0.05834    0.05215    1.12
Trt2        -0.18802    0.05215   -3.61
```

```
Trt3         0.08379    0.05215    1.61

Correlation of Fixed Effects:
     (Intr) Trt1   Trt2
Trt1 -0.353
Trt2 -0.353  0.500
Trt3 -0.353  0.500  0.500
> (fm5Mult <- lmer(Adj ~ 1 + (1|Grp), Multilocation))
Linear mixed model fit by REML
Formula: Adj ~ 1 + (1 | Grp)
   Data: Multilocation
   AIC   BIC logLik deviance REMLdev
 53.33 61.37 -23.66    43.75   47.33
Random effects:
 Groups   Name        Variance Std.Dev.
 Grp      (Intercept) 0.107492 0.32786
 Residual             0.050439 0.22459
Number of obs: 108, groups: Grp, 27

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.85419    0.06669   42.79
> anova(fm2Mult)
Analysis of Variance Table
         Df Sum Sq Mean Sq F value
Location  8 7.3768 0.92210  25.115
Trt       3 1.2217 0.40725  11.092
> (fm2MultR <- lmer(Adj ~ Trt + (Trt - 1|Location) + (1|Block), Multilocatior
+                   verbose = TRUE))
  0:     60.383823: 0.942809 0.942809 0.942809 0.942809  0.00000  0.00000  0.
  1:     26.054753:  1.19385  1.08439  1.12694  1.12034 0.375827 0.404186 0.3
  2:     12.324328:  1.78879  1.10709 0.956709 0.957061 0.834627 0.796973 0.6
  3:     10.066130:  1.94994 0.629726 0.327635 0.637430 0.573542 0.983612  1.
  4:     3.4224559:  1.93818 0.623634 0.292348 0.642854 0.897952 0.873964 0.8
  5:     3.1089777:  1.93984 0.577267 0.242633 0.621572 0.815532 0.971767 0.8
  6:     2.3073226:  1.93441 0.504585 0.160605 0.592976 0.879505 0.901630 0.8
  7:     2.0337629:  1.92971 0.416652 0.0683423 0.559278 0.836635 0.948689 0.
  8:     1.8470590:  1.91328 0.328710  0.00000 0.531012 0.903252 0.915860 0.8
  9:     1.7795236:  1.91237 0.320125  0.00000 0.529888 0.863637 0.939829 0.8
 10:     1.7573420:  1.91169 0.318314 7.08749e-09 0.530015 0.876646 0.928478
```

```
11:     1.7531015:  1.90943 0.310109 0.000128944 0.530270 0.871777 0.935342
12:     1.7391438:  1.90676 0.296234 0.00379742 0.529761 0.871880 0.927261 0
13:     1.7348473:  1.90617 0.295696 0.00295049 0.529191 0.880444 0.932420 0
14:     1.7282649:  1.90334 0.296536 0.00239395 0.528813 0.875925 0.930442 0
15:     1.7197344:  1.89878 0.299011 0.00217862 0.526682 0.881039 0.929714 0
16:     1.7110876:  1.89530 0.303870  0.00000 0.524440 0.872036 0.933236 0.8
17:     1.6879903:  1.88500 0.328108 0.000793068 0.516785 0.870844 0.927959
18:     1.6707757:  1.87983 0.342290 0.000316950 0.506823 0.887923 0.930184
19:     1.6091102:  1.86294 0.376684  0.00000 0.486587 0.862724 0.922704 0.8
20:     1.5702470:  1.84097 0.340493  0.00000 0.443175 0.882584 0.942834 0.8
21:     1.5216217:  1.85580 0.314025 0.0109468 0.402810 0.881588 0.928551 0.
22:     1.4948494:  1.86627 0.319648  0.00000 0.380475 0.870467 0.938187 0.8
23:     1.4929265:  1.86540 0.318133  0.00000 0.383442 0.873084 0.935950 0.8
24:     1.4870920:  1.87047 0.313213  0.00000 0.377164 0.877474 0.930430 0.8
25:     1.4791062:  1.87931 0.306165  0.00000 0.367350 0.872452 0.933874 0.8
26:     1.4763513:  1.88330 0.304699  0.00000 0.357751 0.880229 0.928424 0.8
27:     1.4654401:  1.88784 0.303219  0.00000 0.341859 0.874887 0.932554 0.8
28:     1.4601345:  1.90650 0.273575  0.00000 0.310571 0.870344 0.930091 0.8
29:     1.4581868:  1.90602 0.289675  0.00000 0.307166 0.882478 0.926546 0.8
30:     1.4498468:  1.90588 0.288096  0.00000 0.303214 0.874781 0.932304 0.8
31:     1.4489067:  1.90495 0.286651  0.00000 0.298655 0.874220 0.931639 0.8
32:     1.4484223:  1.90372 0.288931 1.13420e-07 0.293699 0.875148 0.929615
33:     1.4475339:  1.90306 0.289141  0.00000 0.289596 0.873145 0.932112 0.8
34:     1.4472382:  1.90279 0.289569  0.00000 0.288126 0.873782 0.930546 0.8
35:     1.4468920:  1.90268 0.288984  0.00000 0.286710 0.874923 0.932045 0.8
36:     1.4465847:  1.90227 0.289211  0.00000 0.284716 0.873872 0.931105 0.8
37:     1.4439670:  1.89661 0.290547  0.00000 0.263896 0.873166 0.934781 0.8
38:     1.4416697:  1.88986 0.291105 0.00273876 0.243394 0.878146 0.931221 0
39:     1.4416468:  1.89182 0.288849  0.00000 0.234906 0.869593 0.935953 0.8
40:     1.4368350:  1.89124 0.287637  0.00000 0.230224 0.874168 0.933269 0.8
41:     1.4269497:  1.87506 0.271473  0.00000 0.148998 0.877673 0.940364 0.8
42:     1.4143810:  1.87643 0.244212  0.00000 0.105584 0.877020 0.931093 0.8
43:     1.4126447:  1.87047 0.244315  0.00000 0.0805357 0.878274 0.933479 0.
44:     1.4118255:  1.87483 0.236475  0.00000 0.0727321 0.880249 0.938734 0.
45:     1.4107755:  1.87525 0.236907  0.00000 0.0651671 0.880844 0.937370 0.
46:     1.4101442:  1.87763 0.236717  0.00000 0.0575279 0.880201 0.936233 0.
47:     1.4087669:  1.88840 0.238602  0.00000 0.0426940 0.879078 0.934521 0.
48:     1.4078646:  1.89498 0.240044  0.00000 0.0242046 0.877895 0.933632 0.
49:     1.4076240:  1.89698 0.242174 0.00440688 0.0170793 0.877916 0.933798
50:     1.4074080:  1.90248 0.245772  0.00000 0.00128785 0.877399 0.934434 0
```

```
 51:     1.4073389:  1.89458 0.241877  0.00000  0.00000 0.878178 0.934189 0.8
 52:     1.4072193:  1.89841 0.243599  0.00000 0.000711336 0.877797 0.934488
 53:     1.4072108:  1.89750 0.243063  0.00000 0.000678526 0.877937 0.934502
 54:     1.4072098:  1.89742 0.243195 6.06676e-09 0.000672078 0.877973 0.9343
 55:     1.4072091:  1.89741 0.243181 1.13425e-06 0.000649931 0.877959 0.9344
 56:     1.4072084:  1.89742 0.243259 6.08690e-06 0.000584689 0.877961 0.9344
 57:     1.4072073:  1.89704 0.243209  0.00000 0.000190406 0.878020 0.934519
 58:     1.4072072:  1.89690 0.243193  0.00000 5.45575e-05 0.878034 0.934530
 59:     1.4072071:  1.89688 0.243202  0.00000 5.41754e-05 0.878017 0.934503
 60:     1.4072070:  1.89686 0.243194 3.30793e-08 5.34225e-05 0.878021 0.9345
 61:     1.4072068:  1.89696 0.243269  0.00000 0.000272250 0.878003 0.934498
 62:     1.4072064:  1.89678 0.243337  0.00000 0.000411209 0.877991 0.934487
 63:     1.4072057:  1.89677 0.243493  0.00000 0.000776277 0.877984 0.934461
 64:     1.4072051:  1.89655 0.243656  0.00000 0.000434693 0.877993 0.934481
 65:     1.4072044:  1.89665 0.243796 0.000109756 0.000182601 0.877989 0.9344
 66:     1.4072040:  1.89670 0.243785  0.00000 9.08036e-05 0.877999 0.934503
 67:     1.4072040:  1.89670 0.243784  0.00000 6.79441e-06 0.877992 0.934494
 68:     1.4072040:  1.89673 0.243793  0.00000  0.00000 0.877993 0.934500 0.8
 69:     1.4072040:  1.89671 0.243787 8.47225e-06 2.41415e-05 0.877995 0.9344
 70:     1.4072040:  1.89671 0.243786  0.00000 1.05643e-05 0.877993 0.934497
 71:     1.4072040:  1.89671 0.243786  0.00000 9.10919e-06 0.877994 0.934497
 72:     1.4072040:  1.89671 0.243786  0.00000 9.10919e-06 0.877994 0.934497
Linear mixed model fit by REML
Formula: Adj ~ Trt + (Trt - 1 | Location) + (1 | Block)
   Data: Multilocation
   AIC   BIC  logLik deviance REMLdev
 33.41 76.32 -0.7036   -13.38   1.407
Random effects:
 Groups   Name        Variance   Std.Dev.   Corr
 Location Trt1        1.3589e-01 3.6863e-01
          Trt2        1.0700e-01 3.2711e-01 0.989
          Trt3        1.1909e-01 3.4509e-01 0.998 0.996
          Trt4        1.1411e-01 3.3780e-01 0.927 0.972 0.948
 Block    (Intercept) 2.9012e-13 5.3863e-07
 Residual             3.7773e-02 1.9435e-01
Number of obs: 108, groups: Location, 9; Block, 3

Fixed effects:
            Estimate Std. Error t value
(Intercept)  2.86567    0.11865  24.152
```

```
Trt1         0.05834    0.07012   0.832
Trt2        -0.18802    0.05921  -3.176
Trt3         0.08379    0.06447   1.300

Correlation of Fixed Effects:
     (Intr) Trt1   Trt2
Trt1 -0.150
Trt2 -0.306  0.620
Trt3 -0.236  0.682  0.620
```

I PBIB

```
> str(PBIB)
'data.frame':        60 obs. of  3 variables:
 $ response : num  2.4 2.5 2.6 2 2.7 2.8 2.4 2.7 2.6 2.8 ...
 $ Treatment: Factor w/ 15 levels "1","10","11",..: 7 15 1 5 11 13 14 1 2 1 .
 $ Block    : Factor w/ 15 levels "1","10","11",..: 1 1 1 1 8 8 8 8 9 9 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 response ~ Treatment | Block
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      : list()
  ..$ units       : list()
> ## compare with output 1.7  pp. 24-25
> (fm1PBIB <- lmer(response ~ Treatment + (1 | Block), PBIB))
Linear mixed model fit by REML
Formula: response ~ Treatment + (1 | Block)
   Data: PBIB
   AIC   BIC logLik deviance REMLdev
 85.98 121.6 -25.99    22.83   51.98
Random effects:
 Groups   Name        Variance Std.Dev.
 Block    (Intercept) 0.046522 0.21569
 Residual             0.085559 0.29250
Number of obs: 60, groups: Block, 15

Fixed effects:
```

```
             Estimate Std. Error t value
(Intercept)  2.891309   0.166412  17.374
Treatment1  -0.073788   0.222060  -0.332
Treatment10 -0.400249   0.222060  -1.802
Treatment11  0.007392   0.222060   0.033
Treatment12  0.161514   0.222060   0.727
Treatment13 -0.273542   0.222060  -1.232
Treatment14 -0.400000   0.227200  -1.761
Treatment15 -0.032076   0.222060  -0.144
Treatment2  -0.485995   0.222060  -2.189
Treatment3  -0.436366   0.222060  -1.965
Treatment4  -0.107474   0.227200  -0.473
Treatment5  -0.086411   0.222060  -0.389
Treatment6   0.019385   0.222060   0.087
Treatment7  -0.102323   0.222060  -0.461
Treatment8  -0.109705   0.222060  -0.494

Correlation of Fixed Effects:
            (Intr) Trtmn1 Trtm10 Trtm11 Trtm12 Trtm13 Trtm14 Trtm15 Trtmn2
Treatment1  -0.667
Treatment10 -0.667  0.500
Treatment11 -0.667  0.477  0.500
Treatment12 -0.667  0.500  0.500  0.500
Treatment13 -0.667  0.500  0.500  0.500  0.500
Treatment14 -0.683  0.512  0.512  0.512  0.512  0.512
Treatment15 -0.667  0.500  0.477  0.500  0.500  0.500  0.512
Treatment2  -0.667  0.500  0.500  0.500  0.477  0.500  0.512  0.500
Treatment3  -0.667  0.500  0.500  0.500  0.500  0.477  0.512  0.500  0.500
Treatment4  -0.683  0.512  0.512  0.512  0.512  0.512  0.500  0.512  0.512
Treatment5  -0.667  0.500  0.477  0.500  0.500  0.500  0.512  0.477  0.500
Treatment6  -0.667  0.477  0.500  0.477  0.500  0.500  0.512  0.500  0.500
Treatment7  -0.667  0.500  0.500  0.500  0.477  0.500  0.512  0.500  0.477
Treatment8  -0.667  0.500  0.500  0.500  0.500  0.477  0.512  0.500  0.500
            Trtmn3 Trtmn4 Trtmn5 Trtmn6 Trtmn7
Treatment1
Treatment10
Treatment11
Treatment12
Treatment13
Treatment14
```

```
Treatment15
Treatment2
Treatment3
Treatment4   0.512
Treatment5   0.500  0.512
Treatment6   0.500  0.512  0.500
Treatment7   0.500  0.512  0.500  0.500
Treatment8   0.477  0.512  0.500  0.500  0.500
```

J SIMS

```
> str(SIMS)
'data.frame':        3691 obs. of  3 variables:
 $ Pretot: num  29 38 31 31 29 23 23 33 30 32 ...
 $ Gain  : num  2 0 6 6 5 9 7 2 1 3 ...
 $ Class : Factor w/ 190 levels "1","10","100",..: 1 1 1 1 1 1 1 1 1 1 ...
 - attr(*, "ginfo")=List of 7
  ..$ formula     :Class 'formula' length 3 Gain ~ Pretot | Class
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  ..$ order.groups: logi TRUE
  ..$ FUN         :function (x)
  ..$ outer       : NULL
  ..$ inner       : NULL
  ..$ labels      :List of 2
  .. ..$ Pretot: chr "Sum of pre-test core item scores"
  .. ..$ Gain  : chr "Gain in mathematics achievement score"
  ..$ units       : list()
> ## compare to output 7.4, p. 262
> (fm1SIMS <- lmer(Gain ~ Pretot + (Pretot | Class), SIMS))
Linear mixed model fit by REML
Formula: Gain ~ Pretot + (Pretot | Class)
   Data: SIMS
   AIC   BIC logLik deviance REMLdev
 22393 22430 -11190    22373   22381
Random effects:
 Groups   Name        Variance   Std.Dev. Corr
 Class    (Intercept) 14.4894662 3.806503
          Pretot       0.0092027 0.095931 -0.641
 Residual             22.2357583 4.715481
Number of obs: 3691, groups: Class, 190
```

```
Fixed effects:
            Estimate Std. Error t value
(Intercept)   7.0595     0.3659   19.29
Pretot       -0.1860     0.0161  -11.55

Correlation of Fixed Effects:
       (Intr)
Pretot -0.760
```