Remark 1 • *U means a $Uniform(0,1)$ distribution, E is an $Exponential$ distribution, U and E are independent.*

- *S is for a law independent from U such that $P[S=0]=P[S=1]=1/2$.*
- *Z stands for the $Gaussian$ law and G_p represents the $Gamma(1/p,p)$ law.*
- *$Average\ Uniform$ law is also called $Bates(k,a,b)$. In Quesenberry (1977), it is $AveUnif(k+1,0,1)$.*
- *We go from $GeneralizedPareto(\mu,\sigma,\xi)$ to $Pareto(a,k)$ by letting $\mu=k$, $\xi=a^{-1}$ and $\sigma=ka^{-1}$.*
- *We go from $GeneralizedPareto(\mu,\sigma,\xi)$ to a shifted Pareto by letting $\mu=0$, $\xi=1/2$ and $\sigma=1/2$.*
- *We go from $JSU(\mu,\sigma,\nu,\tau)$ to $JSB(g,d)$ by letting $\tau=d$, $\nu=-g$, $\sigma=c^{-1}$ $=\left[(e^{d^2}-1)(e^{d^2}\cosh(2g/d)+1)/2\right]^{-1/2}$ and $\mu=-\sqrt{e^{d^2}}\sinh(g/d)$.*
- *We go from $GED(\mu,\sigma,p)$ to $GED(\lambda)$ by letting $\mu=0$, $p=\lambda$ and $\sigma=\dfrac{1}{\lambda^{1/\lambda}\sigma}$ with $C_\lambda=\sqrt{\Gamma(3\lambda^{-1})/\Gamma(\lambda^{-1})}$.*
- *Variance of $VUnif(j)$ is given by:*

$$\begin{aligned}\mathbb{V}ar(Y_j) =& \frac{1}{12(j+1)} - \frac{1}{4} + \frac{1}{(j+1)!}\sum_{k=0}^{j+1}(-1)^k \binom{j+1}{k} * \\ & \left\{(-1)^{j+1}\frac{k^{j+2}}{(j+1)(j+2)} - sign(k-\frac{j+1}{2})\left(\frac{j+1}{2}-k\right)^{(j+1)}\left[\frac{1}{j+2}\left(\frac{j+1}{2}-k\right)+\frac{k}{j+1}\right]\right\}.\end{aligned}$$

where $sign(0)=-1$.

Table 1: Probability distributions

	Law	Notation	Density	Generation	Expectation	Variance
1	Laplace	$Lp(\mu, b)$	$\frac{1}{2b}\exp\left(-\frac{\lvert x-\mu\rvert}{b}\right)$	$\mu - b.sgn\{U - \frac{1}{2}\}\ln(1 - 2\lvert U - \frac{1}{2}\rvert)$	μ	$2b^2$
2	Normal	$N(\mu, \sigma)$	$(\sqrt{2\pi}\sigma)^{-1}\exp^{-\frac{x^2}{2\sigma^2}}$	$\sigma Z + \mu$	μ	σ^2
3	Cauchy	$Cauchy(l, s)$	$\frac{1}{\pi s(1+(\frac{x-l}{s})^2)}$	rcauchy(location,scale)	undefined	undefined
4	Logistic	$Lg(\mu, s)$	$\frac{1}{s}e^{-\frac{x-\mu}{s}}(1 + e^{-\frac{x-\mu}{s}})^{-2}$	$\mu + s\ln\left(\frac{U}{1-U}\right)$	μ	$\frac{\pi^2}{3}s^2$
5	Gamma	$Gamma(a, b)$	$\frac{1}{(1/b)^a\Gamma(a)}x^{a-1}e^{-xb}, X \geq 0, a, b > 0$	rgamma(a,1/b)	$\frac{a}{b}$	$\frac{a}{b^2}$
6	Beta	$Beta(\alpha, \beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, 0 \leq x \leq 1$	rbeta(α, β)	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
7	Uniform	$U(a, b)$	$(b-a)^{-1}, a \leq x \leq b$	$\frac{U}{(b-a)} + a$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
8	Student	$Student - t(k)$	$(\sqrt{k\pi})^{-1}\frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}\left(1 + \frac{x^2}{k}\right)^{-\frac{k+1}{2}}, k > 0$	$\frac{Z}{\sqrt{\chi^2(k)/k}}$	k=1: undefined k¿1: 0	$k \leq 2: \quad \infty$ $k > 2: \quad \frac{k}{k-2}$
9	Chi-squared	$\chi^2(k)$	$2^{-k/2}\Gamma(k/2)^{-1}x^{k/2-1}e^{-x/2}, x > 0$	$\sum_{i=1}^{k} Z_i^2$	k	$2k$
10	Log Normal	$LN(\mu, \sigma)$	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$	$\exp\{N(\mu, \sigma^2)\}$	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$
11	Weibull	$W(\lambda, k)$	$\frac{\lambda}{k}\left(\frac{x}{k}\right)^{\lambda-1}e^{-(x/k)^\lambda}$	$k(-\ln(U))^{1/\lambda}$	$\mu = k\Gamma\left(1 + \lambda^{-1}\right)$	$k^2\Gamma\left(1 + \frac{2}{\lambda}\right) - \mu^2$
12	Shifted Exponential	$SE(l, b)$	$b\exp\{-(x-l)b\}, x \geq l$	$\frac{-\ln U}{b} + l$	$l + \frac{1}{b}$	$\frac{1}{b^2}$
13	Power Uniform	U^{1+j}	$\frac{1}{1+j}x^{-\frac{j}{j+1}}$	U^{1+j}	$\frac{1}{j+2}$	$\frac{1}{2j+3}\frac{(j+1)^2}{(j+2)^2}$
14	Average Uniform	$AveUnif(k, a, b)$	$\frac{k^k}{(k-1)!}\sum_{j=0}^{\lfloor k\frac{x-a}{b-a}\rfloor}(-1)^j\binom{k}{j}(\frac{x-a}{b-a} - \frac{j}{k})^{k-1}$ for $a \leq x \leq b$	mean(runif(k,a,b))	$\frac{1}{2}(a+b)$	$\frac{1}{12k}(b-a)^2$
15	UUniform	$UUnif(j)$	$(2(1+j))^{-1}(x^{-j/(1+j)} + (1-x)^{-j/(1+j)})$	$SU^{j+1} + (1-S)(1-U^{j+1})$	$\frac{1}{2}$	$\frac{2j^2+3j+2}{2(2j+3)(2j+4)}$
16	VUniform	$VUnif(j)$	$f_{14}(x - \frac{1}{2})\mathbb{I}\{x < 1\} + f_{14}(x + \frac{1}{2})\mathbb{I}\{x \geq 0\}$ where f_{14} is $AveUnif(j+1, 0, 1)$	if $Z_{j+1} < 0.5$: $Z_{j+1} + 0.5$, else: $Z_{j+1} - 0.5$, with $Z_{j+1} = AveUnif(j+1)$	$\frac{1}{2}$	see Remark
17	Johnson SU	$JSU(\mu, \sigma, \nu, \tau)$	$\frac{1}{c\sigma\frac{1}{\tau}}\frac{1}{\sqrt{z^2+1}}\frac{1}{\sqrt{2\pi}}e^{-r^2/2}$	$\mu + c\sigma\sqrt{w}\sinh(\omega) + c\sigma\sinh\left(\frac{1}{\tau}(Z+\nu)\right)$ $r = -\nu + \tau\sinh^{-1}(z)$ $z = \frac{x-(\mu+c\sigma\sqrt{w}\sinh(\omega))}{c\sigma}$ $c = ((w-1)(w\cosh(2\omega)+1)/2)^{-1/2}$ $w = e^{(\frac{1}{\tau})^2}$ and $\omega = -\nu\frac{1}{\tau}$	μ	σ^2
18	Symmetrical Tukey	$TU(l)$	undefined	$\frac{U^l-(1-U)^l}{l}, -1 \leq X \leq 1$	0	$\frac{2}{l^2}\left(\frac{1}{2l+1} - \frac{\Gamma^2(l+1)}{\Gamma(2l+2)}\right)$
19	Location Contaminated	$LoConN(p, m)$	$\frac{1}{\sqrt{2\pi}}\left[pe^{-\frac{(x-m)^2}{2}} + (1-p)e^{-\frac{x^2}{2}}\right]$	U = runif(0,1); if (U¡p) x = rnorm(m,1) ; otherwise x = rnorm(0,1)	pm	$1 - (pm)^2 + pm^2$

following on next page

Table 1 – continuation from previous page

	Law	Notation	Density	Generation	Expectation	Variance
20	Johnson SB	$JSB(g,d)$	$\frac{d}{\sqrt{2\pi}}\frac{1}{x(1-x)}e^{-\frac{1}{2}\left(g+d\ln\frac{x}{1-x}\right)^2}, d>0$	$\left(1+e^{-\frac{Z-g}{d}}\right)^{-1}, 0<X<1$	undefined	undefined
21	Skew Normal	$SkewN(\xi,\omega,\alpha)$	$\left(\frac{2}{\omega}\right)\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\left(\frac{x-\xi}{\omega}\right)\right), \omega>0$	$\xi+\omega Y$ if $(U_0\geq 0)$ $Y=U_1$; otherwise $Y=-U_1$ U_0, V independent of $N(0,1)$ $U_1=\delta U_0+\sqrt{1-\delta^2}V$ $\delta=\alpha/\sqrt{1+\alpha^2}$	$\xi+\omega\sqrt{2/\pi}\delta$	$\omega^2(1-2\delta^2/\pi)$
22	Scale Contaminated	$ScConN(p,d)$	$\frac{1}{\sqrt{2\pi}}\left[\frac{p}{d}e^{-\frac{x^2}{2d^2}}+(1-p)e^{-\frac{x^2}{2}}\right]$	U = runif(0,1); if (U;p) x = rnorm(0,d) ; otherwise x = rnorm(0,1)	0	pd^2+1-p
23	Generalized Pareto	$GP(\mu,\sigma,\xi)$	si $\xi>0$: $1\!\!1[0\leq x\leq\mu+\frac{\sigma}{\xi}]\frac{1}{\sigma}(1-\xi\frac{x-\mu}{\sigma})^{(1-\xi)/\xi}$ else : $1\!\!1[-\mu\leq x\leq\infty]\frac{1}{\sigma}(1-\xi\frac{x-\mu}{\sigma})^{(1-\xi)/\xi}$	$\mu-\frac{\sigma(U^{\xi}-1)}{\xi}$	$\mu+\frac{\sigma}{1+\xi}$ $(\xi<1)$	$\frac{\sigma^2}{(1+\xi)^2(1+2\xi)}$ $(\xi<1/2)$
24	Generalized Error Distribution	$GED(\mu,\sigma,p)$	$\frac{p}{2\sigma\Gamma(1/p)}e^{-(\lvert x-\mu\rvert/\sigma)^p}$	$\mu+\sigma\left(\frac{G_p}{p}\right)^{(1/p)}sign(U-1/2)$	μ	$\frac{\sigma^2\Gamma(3/p)}{\Gamma(1/p)}$
25	Stable	$S(\alpha,\beta,c,\mu)$	undefined $0<\alpha\leq 2, -1\leq\beta\leq 1$ $c>0$ et $\mu\in\mathbb{R}$	si ($\alpha=1$ et $\beta=0$) tmp = rcauchy(0,1) $x=tmp*c+\mu$; else se function **rstable()** in package **stabledist**	μ if $\alpha>1$ undefined otherwise	$2c^2$ if $\alpha=2$ ∞ otherwise
26	Gumbel	$Gumbel(\mu,\sigma)$	$\frac{1}{\sigma}\exp\left\{-\exp\left[-\left(\frac{x-\mu}{\sigma}\right)\right]-\left(\frac{x-\mu}{\sigma}\right)\right\}$	$\mu-\sigma\ln(E)$	$\mu+\sigma(-\Gamma'(1))$	$\frac{\pi^2}{6}\sigma^2$
27	Frechet	$Frechet(\mu,\sigma,\alpha)$	$\frac{\alpha}{\sigma}\left(\frac{x-\mu}{\sigma}\right)_+^{-\alpha-1}\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right\}$	$\mu+\sigma E^{-1/\alpha}$	si $\alpha>1$: $\mu+\sigma\Gamma(1-\frac{1}{\alpha})$; else ∞	si $\alpha>2$: $\sigma^2(\Gamma(1-\frac{2}{\alpha})$ $-(\Gamma(1-\frac{1}{\alpha}))^2)$; else ∞
28	Generalized Extreme Value	$GEV(\mu,\sigma,\xi)$	$\xi\neq 0$: $[1+z]^{-\frac{1}{\xi}-1}\exp\left\{-[1+z]^{-\frac{1}{\xi}}\right\}/\sigma$ with $z=\xi\frac{x-\mu}{\sigma}$, for $1+z>0$ $\xi=0$: Gumbel	if $\xi=0$: $\mu-\sigma\ln(E)$ else: $\mu+\sigma(E^{-\xi}-1)/\xi$	si $\xi\neq 0, \xi<1$: $\mu+\sigma\frac{\Gamma(1-\xi)-1}{\xi}$; $\mu+\sigma\gamma$ if $\xi=0$; ∞ if $\xi\geq 1$; γ: Euler constant	if $\xi\neq 0, \xi<\frac{1}{2}$: $\sigma^2\frac{(g_2-g_1^2)}{\xi^2}$; $\sigma^2\frac{\pi^2}{6}$ if $\xi=0$; ∞ if $\xi\geq\frac{1}{2}$; $g_k=\Gamma(1-k\xi)$
29	Generalized Arcsine	$GArcSine(\alpha)$	$\frac{\sin(\pi\alpha)}{\pi}x^{-\alpha}(1-x)^{\alpha-1}$ for $0\leq x\leq 1$ et $0<\alpha<1$	$rbeta(1-\alpha,\alpha)$	$1-\alpha$	$(1-\alpha)\alpha/2$
30	Folded Normal	$FoldN(\mu,\sigma)$	dnorm(x,mu,sigma)+dnorm(-x,mu,sigma) for $x\geq 0$	$\lvert N(\mu,\sigma^2)\rvert$	$\sigma\sqrt{\frac{2}{\pi}}e^{-\frac{\mu^2}{2\sigma^2}}$ $+\mu[1-2\Phi(-\frac{\mu}{\sigma})]$	$\mu^2+\sigma^2-$ $\left\{\sigma\sqrt{\frac{2}{\pi}}e^{-\frac{\mu^2}{2\sigma^2}}+\dots\right\}$

following on next page

Table 1 – continuation from previous page

	Law	Notation	Density	Generation	Expectation	Variance
						$\left\{ \ldots + \mu[1 - 2\Phi(-\frac{\mu}{\sigma})] \right\}^2$
31	Mixture Normal	$MixN(p, m, d)$	p*dnorm(x,m,d)+(1-p)*dnorm(x)	U = runif(0,1); si (U¡p) x = rnorm(m,d); sinon x[i] = rnorm(0,1)	mp	$(1-p)(1+pm^2)+pd^2$
32	Truncated Normal	$TruncN(a, b)$	$\frac{\exp(-x^2/2)}{\sqrt{2\pi}(\Phi(b)-\Phi(a))} \mathbb{I}[a \leq x \leq b]$	Z = rnorm(0,1) while ((Z¡a) —— (Z¿b)) Z = rnorm(0,1) x = Z	$\frac{\phi(a)-\phi(b)}{\Phi(b)-\Phi(a)}$	$1 + \frac{a\phi(a)-b\phi(b)}{\Phi(b)-\Phi(a)}$ $- \left(\frac{\phi(a)-\phi(b)}{\Phi(b)-\Phi(a)}\right)^2$
33	Normal with outliers	$Nout(a)$	undefined	$\mathbf{a} \in \{1, 2, 3, 4, 5\}$ x = rnorm(0,1) with **a** outliers	0	1
34	Generalized Exponential Power	$GEP(t1, t2, t3)$	if $\lvert x\rvert \geq z_0$: $p(x; \gamma, \delta, \alpha, \beta, z_0) \propto$ $e^{-\delta\lvert x\rvert^{\gamma}} \lvert x\rvert^{-\alpha} (log\lvert x\rvert)^{-\beta}$ si $\lvert x\rvert < z_0$: $p(x; \gamma, \delta, \alpha, \beta, z_0)$	see function **law34.cpp** in **PoweR**	undefined	undefined
35	Exponential	$Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$	rexp($\frac{1}{\lambda}$)	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
36	Asymmetric Laplace	$ALp(\mu, b, k)$	for $x \leq \mu$: $f(x) = \frac{\sqrt{2}}{b} \frac{k}{1+k^2} exp\left(-\frac{\sqrt{2}}{bk}\lvert x-\mu\rvert\right)$ for $x > \mu$: $f(x) = \frac{\sqrt{2}}{b} \frac{k}{1+k^2} exp\left(-\frac{\sqrt{2}k}{b}\lvert x-\mu\rvert\right)$	$\mu + b\log(\frac{runif(n)^k}{runif(n)^{1/k}})/\sqrt{2}$	$\mu + b * \frac{(\frac{1}{k} - k)}{\sqrt{2}}$	$b^2 \frac{1+k^4}{2k^2}$
37	Normal-inverse Gaussian	$NIG(\alpha, \beta, \delta, \mu)$	$\frac{\alpha\delta K_1(\alpha\sqrt{\delta^2+(x-\mu)^2})}{\pi\sqrt{\delta^2+(x-\mu)^2}} e^{\delta\gamma+\beta(x-\mu)}$ $\gamma = \sqrt{\alpha^2 - \beta^2}$ K_1: Bessel function of the second kind	see **rnig()** in package **fBasics**	$\mu + \frac{\beta\delta}{\gamma}$	$\frac{\delta\alpha^2}{\gamma^3}$
38	Asymmetric Power Distribution	$APD(\theta, \phi, \alpha, \lambda)$	dens	gen	$\theta + \frac{\Gamma\left(\frac{2}{\lambda}\right)}{\Gamma\left(\frac{1}{\lambda}\right)} (1-2\alpha)\delta^{-\frac{1}{\lambda}}$ $\delta = \frac{2\alpha^{\lambda}(1-\alpha)^{\lambda}}{\alpha^{\lambda}+(1-\alpha)^{\lambda}}$	$\phi^2[\Gamma\left(\frac{3}{\lambda}\right)\Gamma\left(\frac{1}{\lambda}\right)(1-3\alpha+3\alpha^2)$ $-\Gamma^2\left(\frac{2}{\lambda}\right)(1-2\alpha)^2]/[\Gamma^2\left(\frac{1}{\lambda}\right)\delta^{\frac{2}{\lambda}}]$